NETWORKS

CHAPTER- I NFTL(III SEM)

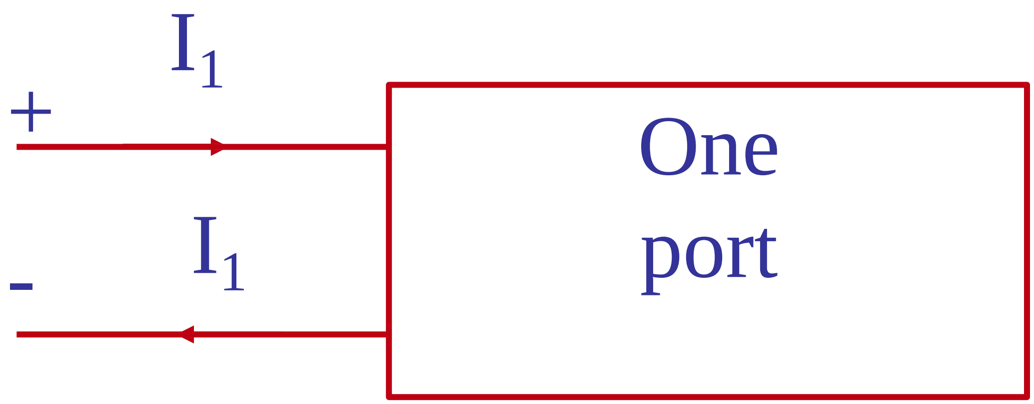
By

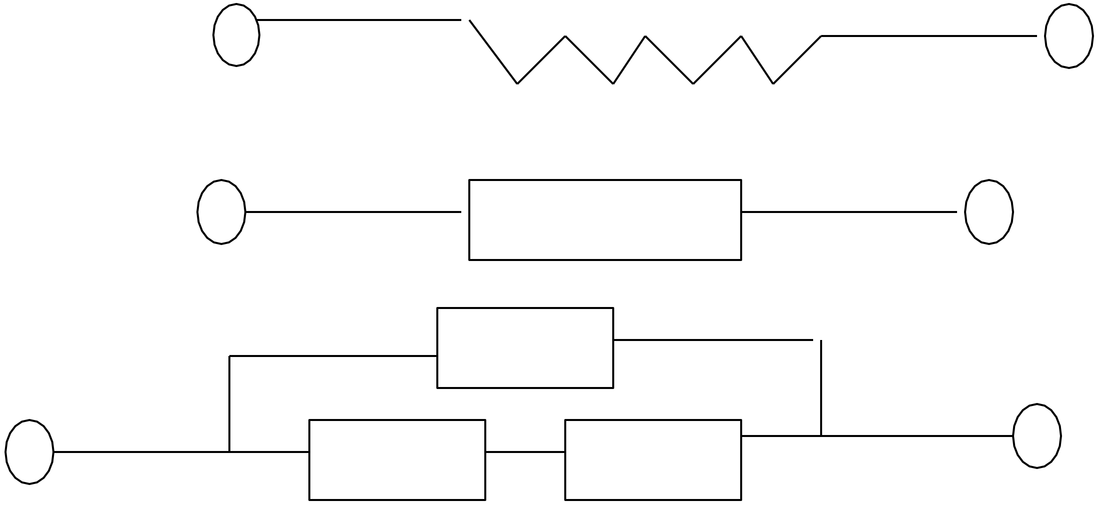
Mr. Rajesh Kumar Singla

Lecturer in ECE

Govt. Polytechnic Sirsa

ONE PORT NETWORK

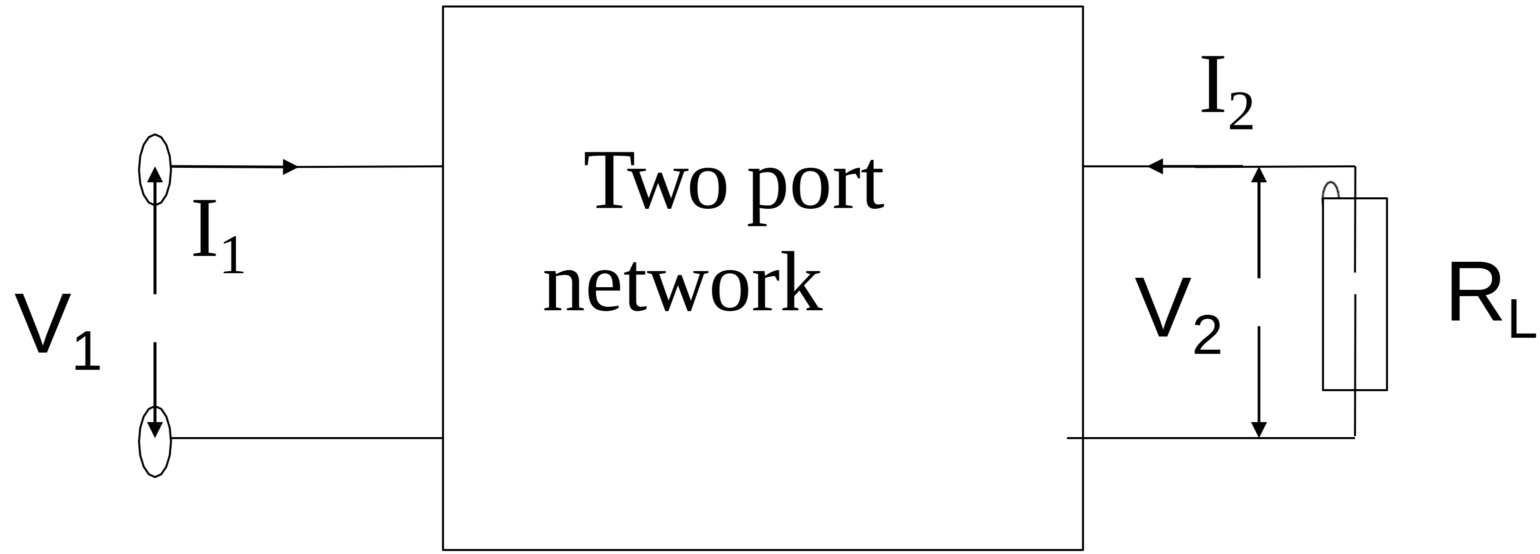




TWO PORT NETWORK

- A Network having two pair of terminals .
- Two port networks act as building blocks of electrical or electronic circuits.
- One pair of terminal is connected to energy source and other pair of terminals is connected to load.
- A transmission line ,an amplifier, a transformer are examples of two port n/w.

TWO PORT NETWORK



V_1, V_2 : Voltages at i/p & o/p ports

I_1, I_2 : Currents at i/p & o/p ports

Out of four variables,only two are dependent variables.

TWO PORT NETWORK

- The dependence of two of four variables of two port n/w can be expressed in number of ways with the help of parameters.

- An important aspect of two port representation is that all the parameters can be obtained from measurements made at the i/p &o/p ports.

TWO PORT NETWORK

- The internal cktry of the n/w need not known.
- Often the circuit between the two ports is highly complex. These networks are linear and passive and may contain controlled sources but not independent sources inside.

TWO PORT NETWORK

- The two port parameters provide a shorthand method for analyzing the input-output properties of two ports without having to deal directly with the highly complex circuit internal to the two port.

- A two-port network makes possible the isolation of either a complete circuit or part of it and replacing it by its characteristic parameters.

NETWORK ELEMENTS

- LINEAR & NONLINEAR: Elements following linear relationship between V & I,known as linear elements otherwise nonlinear.
- UNILATERAL& BILATERAL: If the magnitude of the current passing through that element is affected due to change in polarity of the applied voltage then known as unilateral element

e.g.Diode,Transistors otherwise bilateral e.g R,L & C.

Network Elements

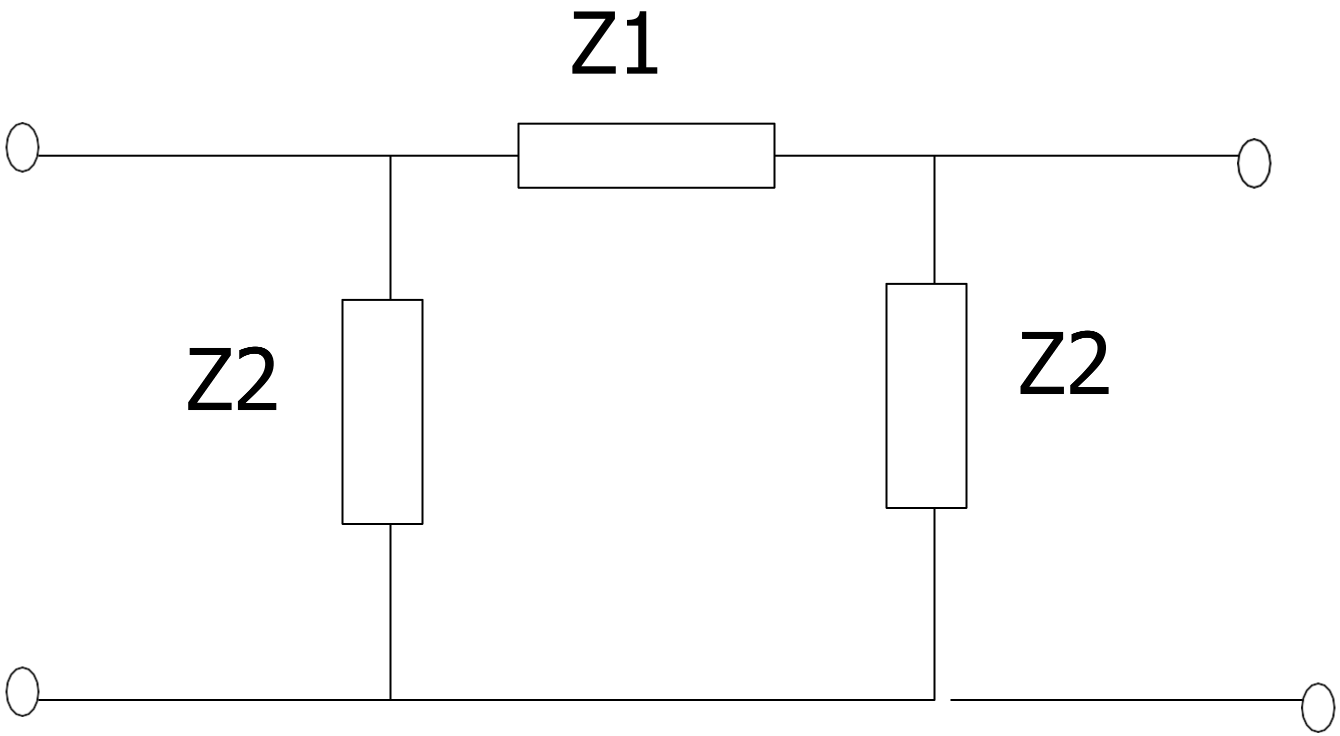
ACTIVE &PASSIVE ELEMENTS

- If a circuit element has the capabilities of processing or enhancing the energy level of a signal passing through it,is called an active element e.g.transistor,opamp etc.
- Resistors, inductors&capicitors etc. are passive as they don't have any intrinsic means of signal boosting.

CLASSIFICATION OF NETWORK

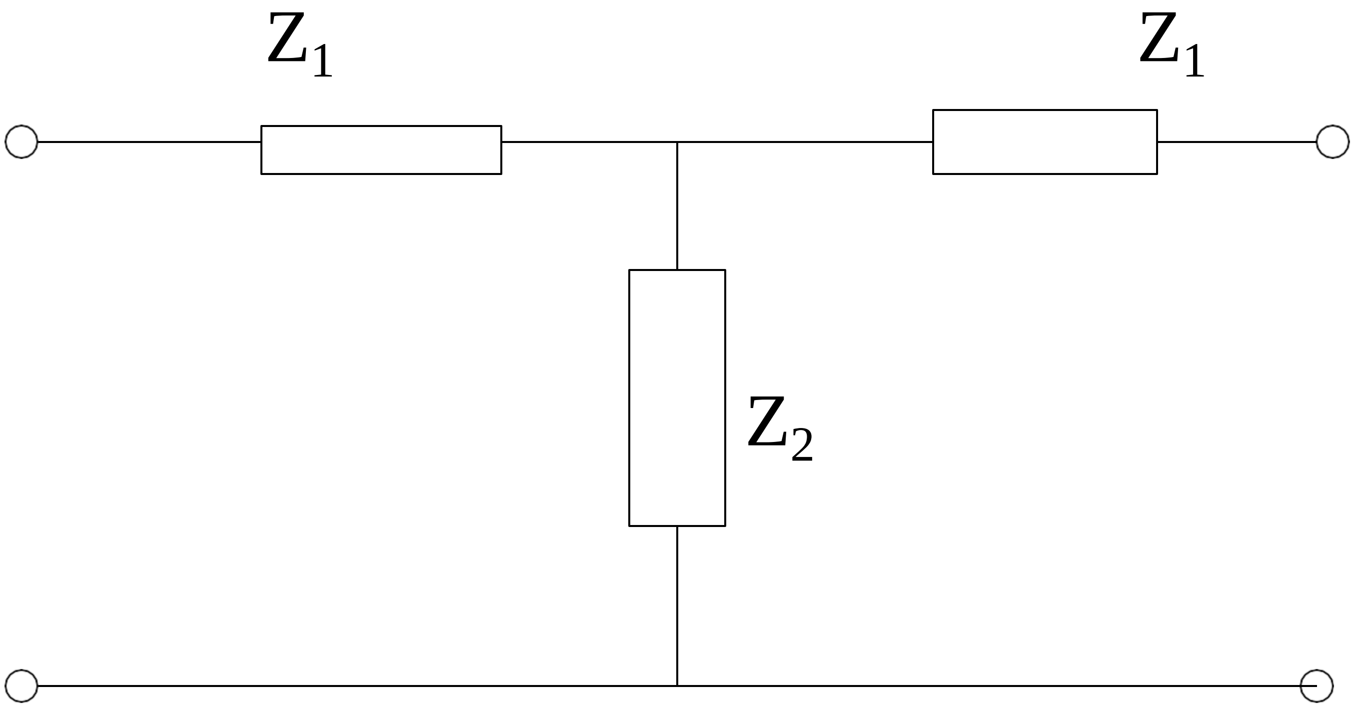
- **SYMMETRICAL NETWORK:**
  - It is one whose electrical characteristics do not change when its input and output terminals are interchanged.
- Symmetrical network offers same impedance at the input and output terminals.

CLASSIFICATION OF NETWORK



SYMMETRICAL π NETWORK

CLASSIFICATION OF NETWORK



SYMMETRICAL T-NETWORK

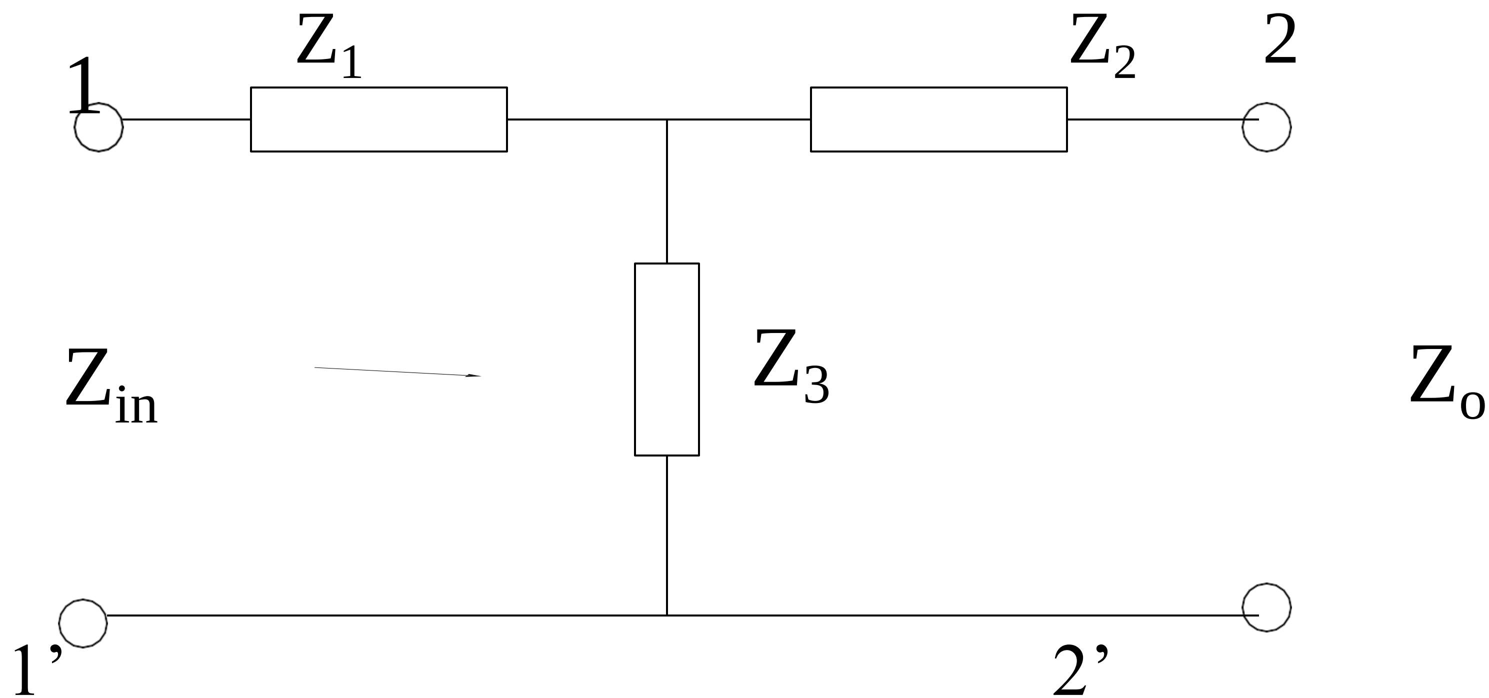
CLASSIFICATION OF NETWORK

ASYMMETRICAL NETWORK

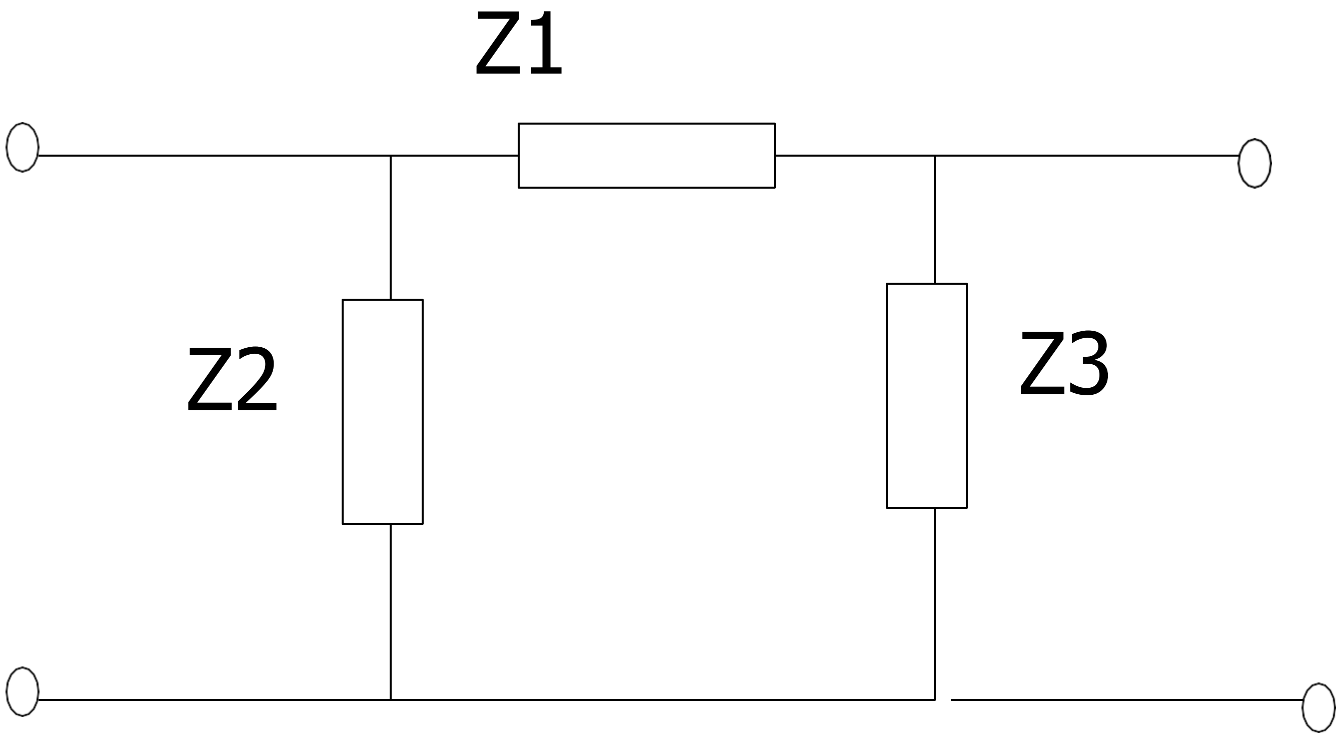
- It is one whose electrical characteristics changes when its input and output terminals are interchanged.
- Asymmetrical network doesn't offer same impedance at the input and output terminals.

CLASSIFICATION OF NETWORK

Asymmetrical T-Networks:



CLASSIFICATION OF NETWORK



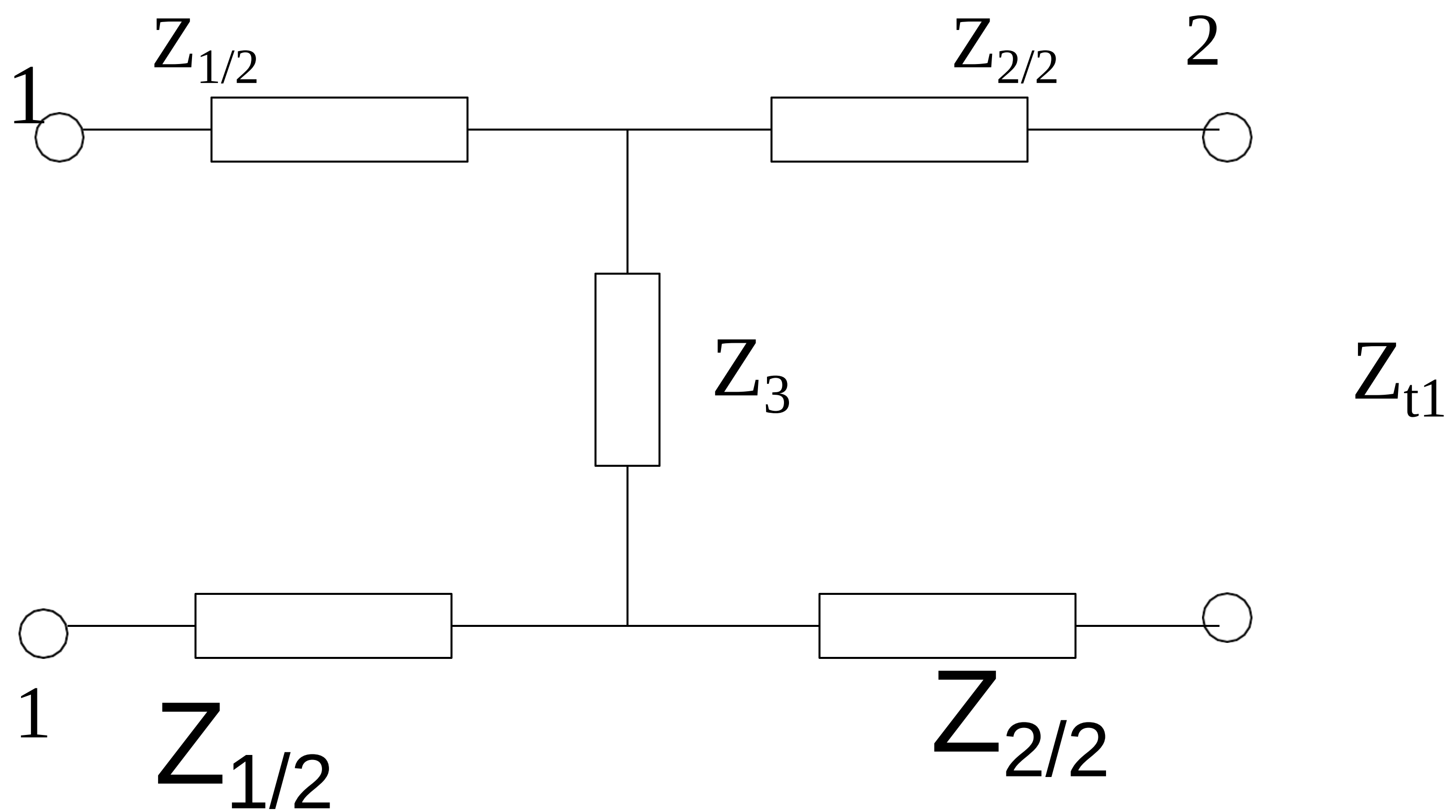
Asymmetrical π Network

CLASSIFICATION OF NETWORK

BALANCED NETWORK

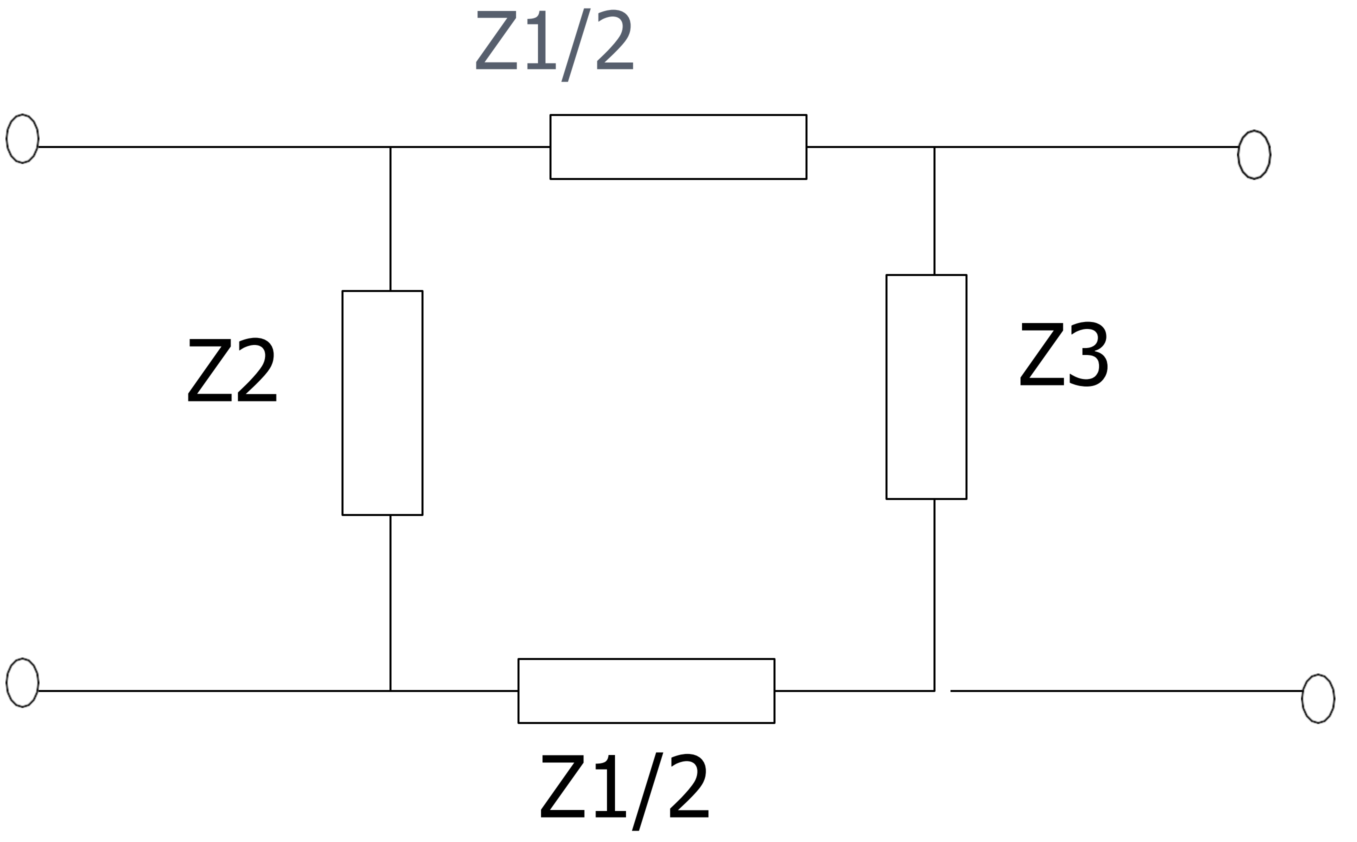
- It is one , in which all the series impedances are identical/same and also these are symmetrical with respect to ground.Thus,the corresponding series arm impedances must be equal.

CLASSIFICATION OF NETWORK



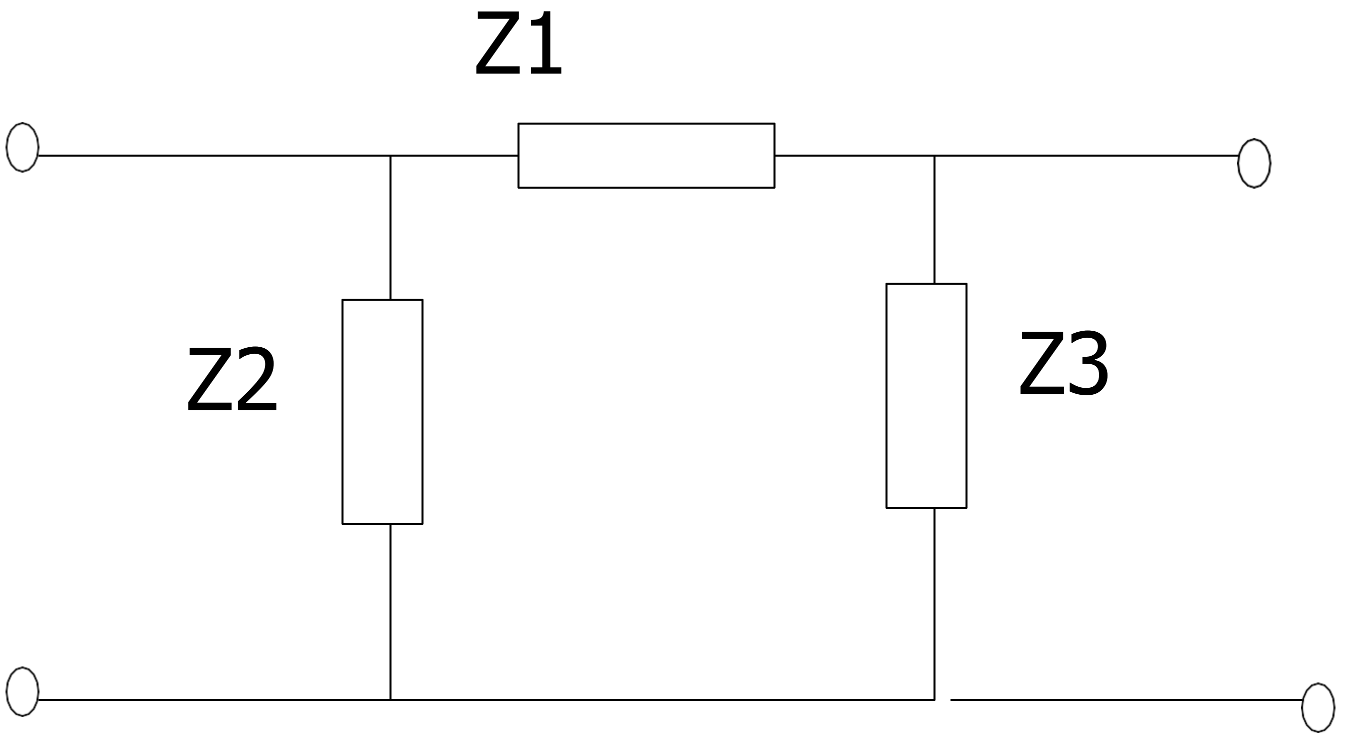
BALANCED T N/W

CLASSIFICATION OF NETWORK



BALANCED π- Network

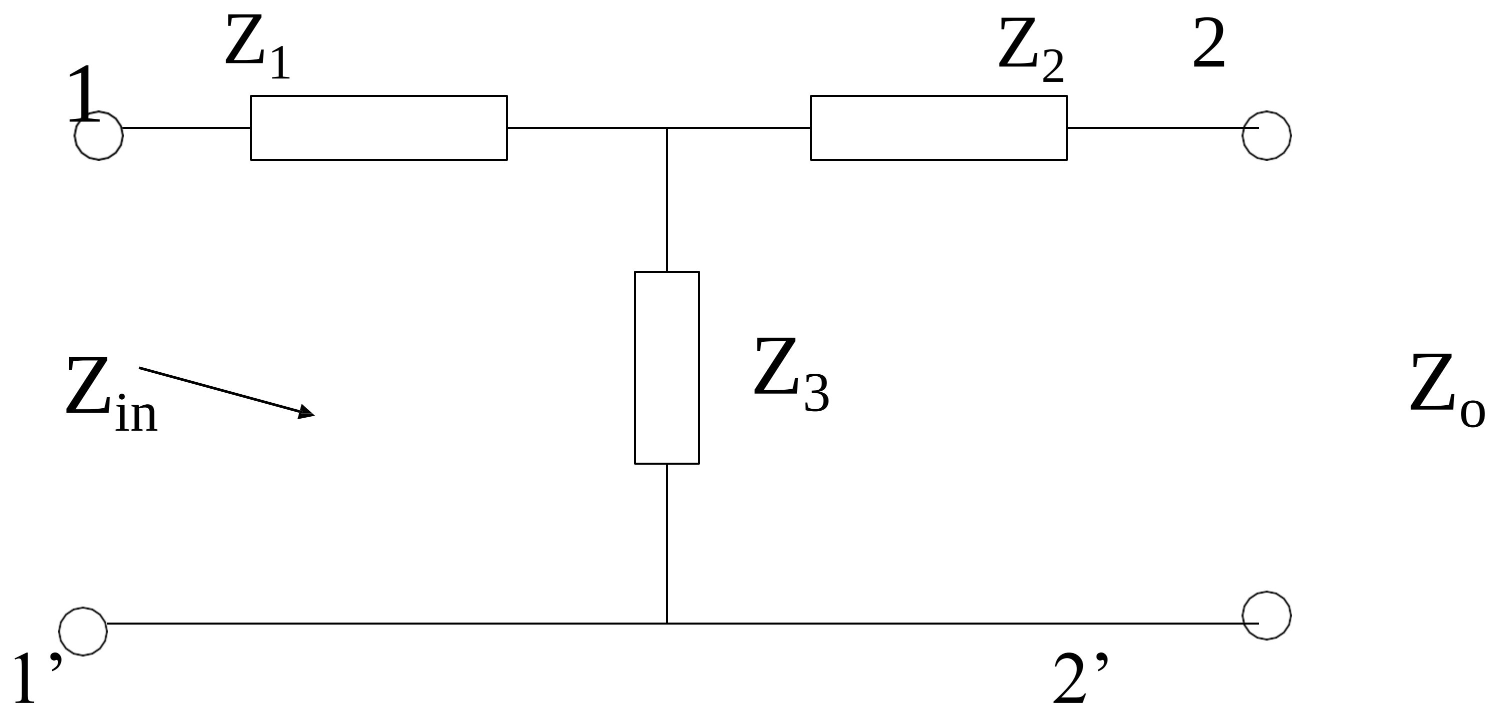
CLASSIFICATION OF NETWORK



unbalanced π Network

CLASSIFICATION OF NETWORK

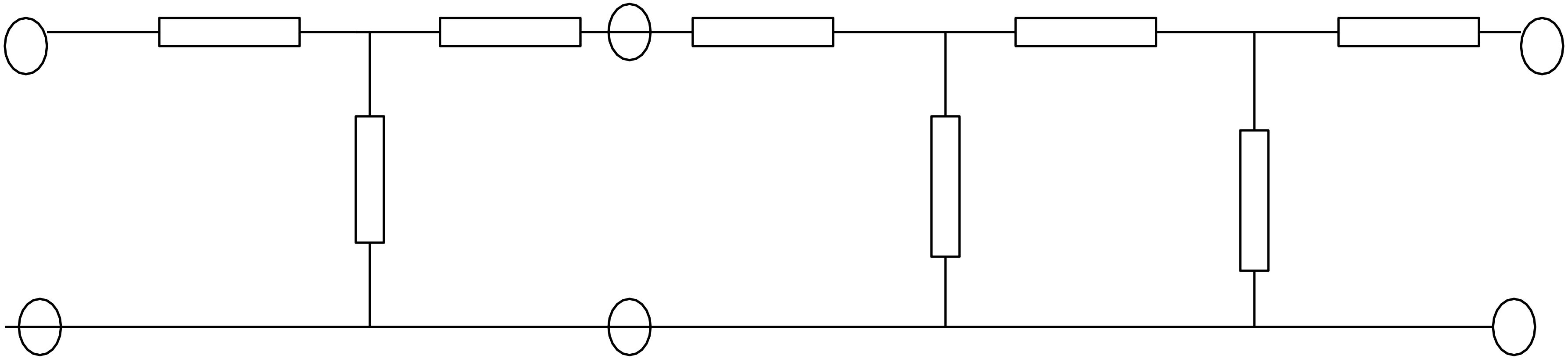
UNBALANCED N/W



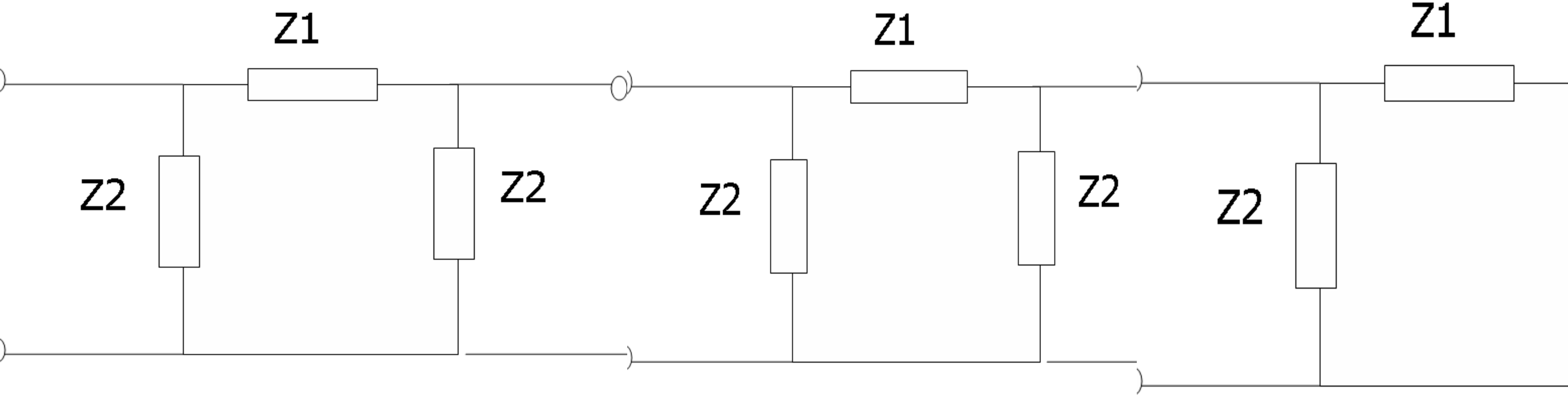
LADDER NETWORK

It is one which consists of a large number of similar networks connected one after other. A ladder network may be balanced network or unbalanced network .

LADDER NETWORK ,T TYPE



LADDER NETWORK ,π TYPE



NETWORK CONFIGURATION

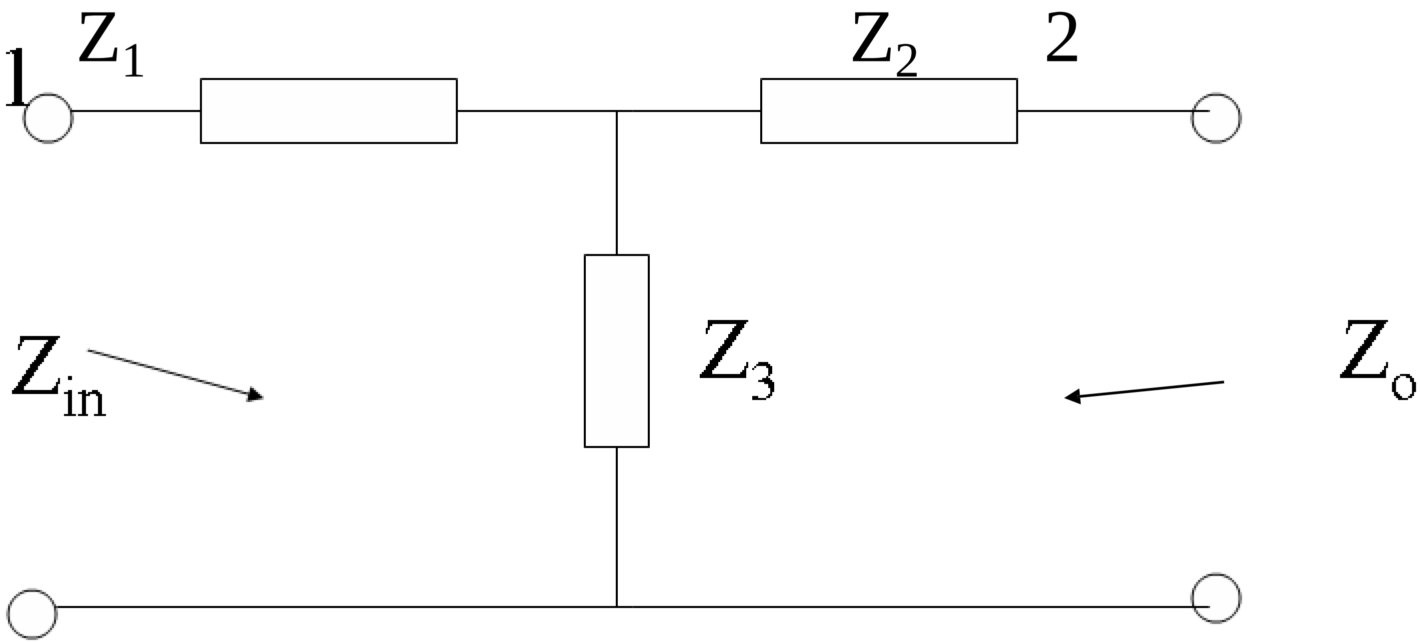
In any two port network, the impedance can be configured together to acquire different shapes, commonly known as the configuration of network.

1. T-section.
2. π-section.
3. Half Section.
4. L-section
5. Lattice Section.
6. Bridge T-section

T-SECTION

A network that looks like alphabet ‘T’ is called T-section. T section can be symmetrical and asymmetrical. Both the series impedance are same in symmetrical T network.

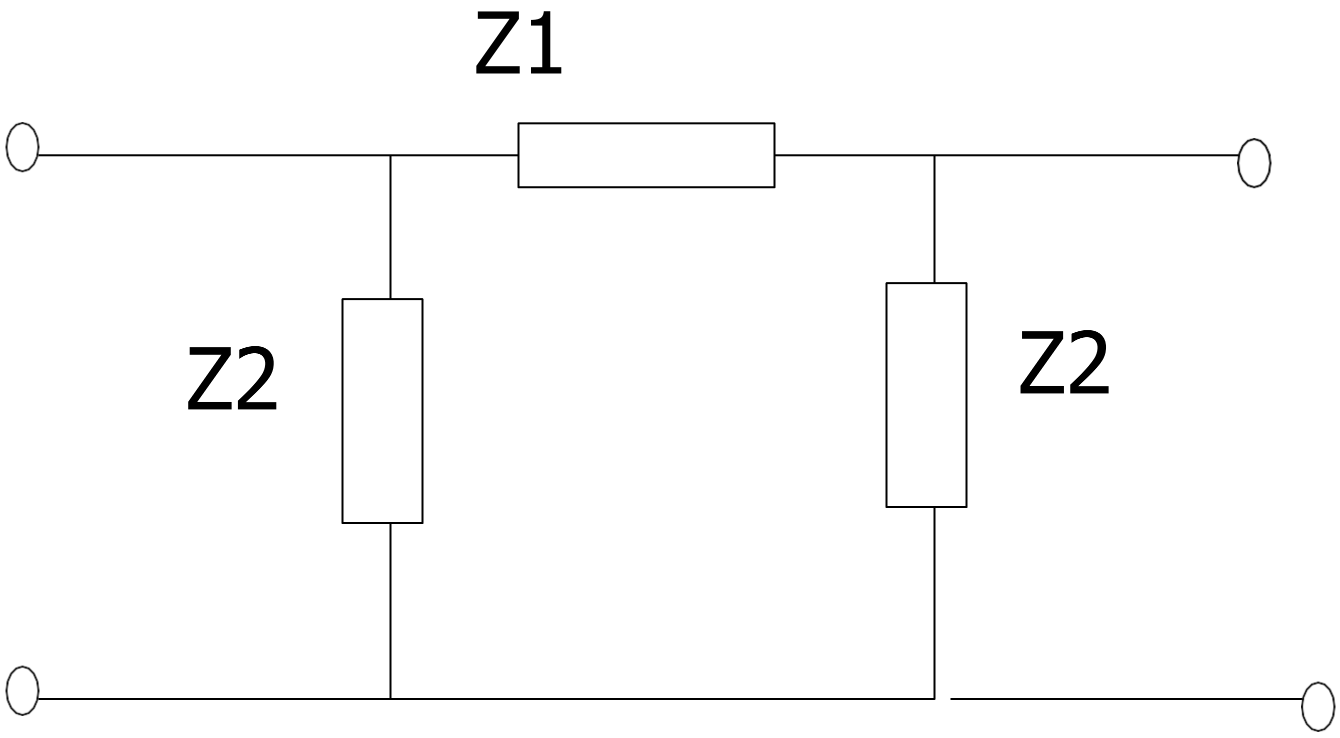
Asymmetrical T-Networks



π-SECTION

The network section which looks like ‘π’ is called π-section. π-section can be symmetrical and asymmetrical. In symmetrical π-section, both the shunt impedance are same.

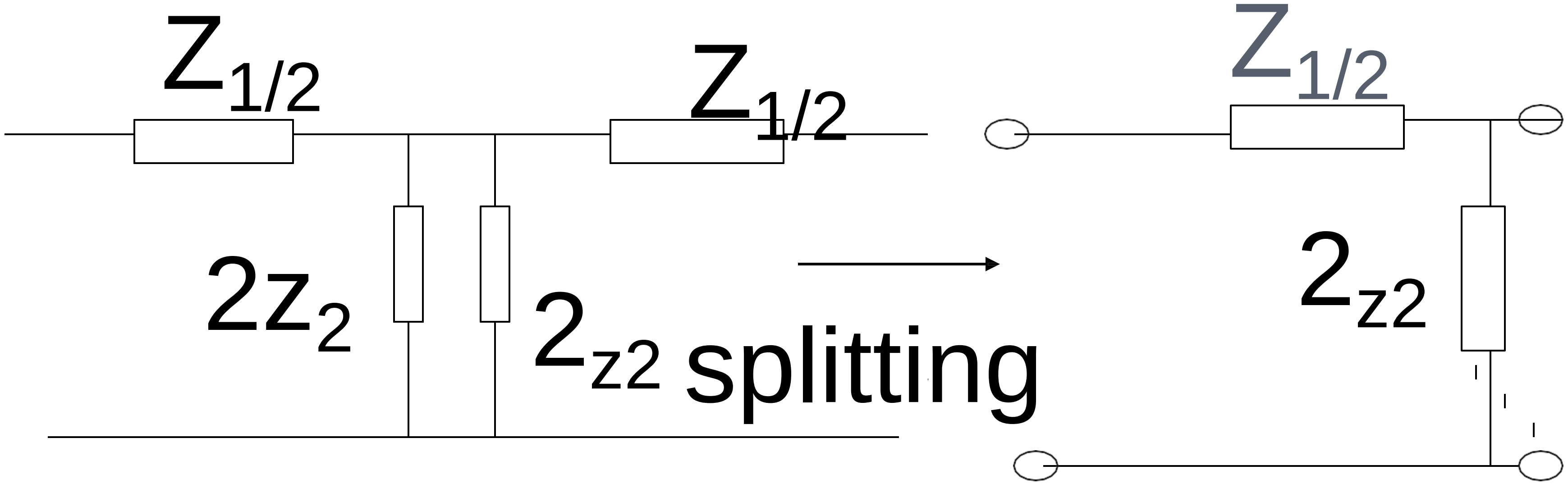
SYMMETRICAL π NETWORK



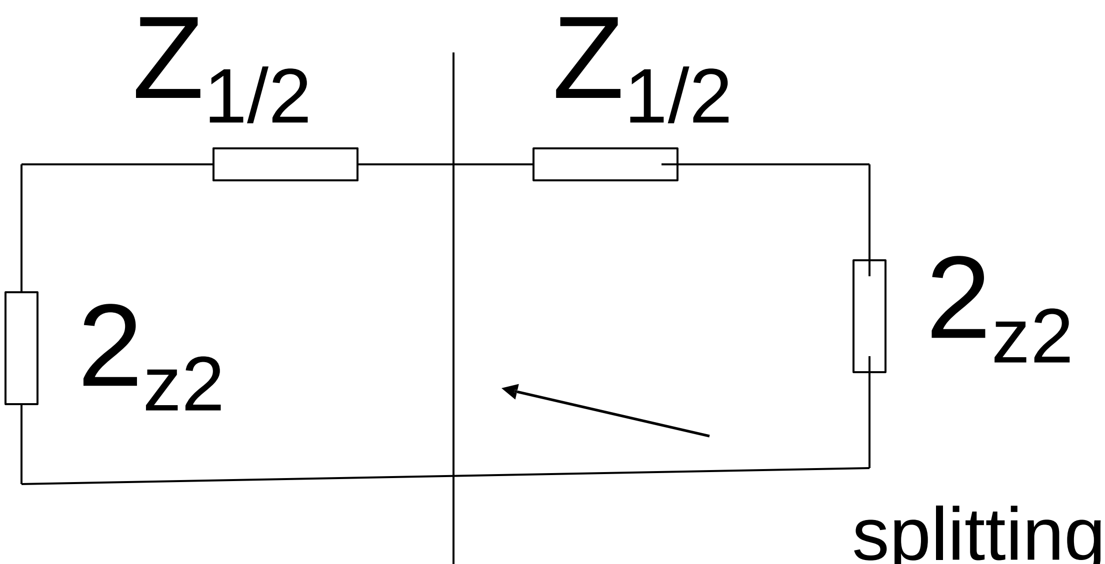
Half Section

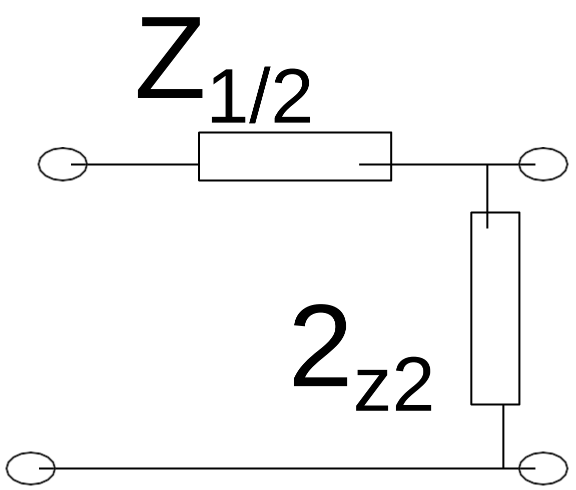
A half section is used for impedance maching purpose. A half section is obtained by splitting symmetrical T or π sections into two halves. Each half is known as half section. Half section obtained is asymmetrical in nature.

Half Section



Half Section

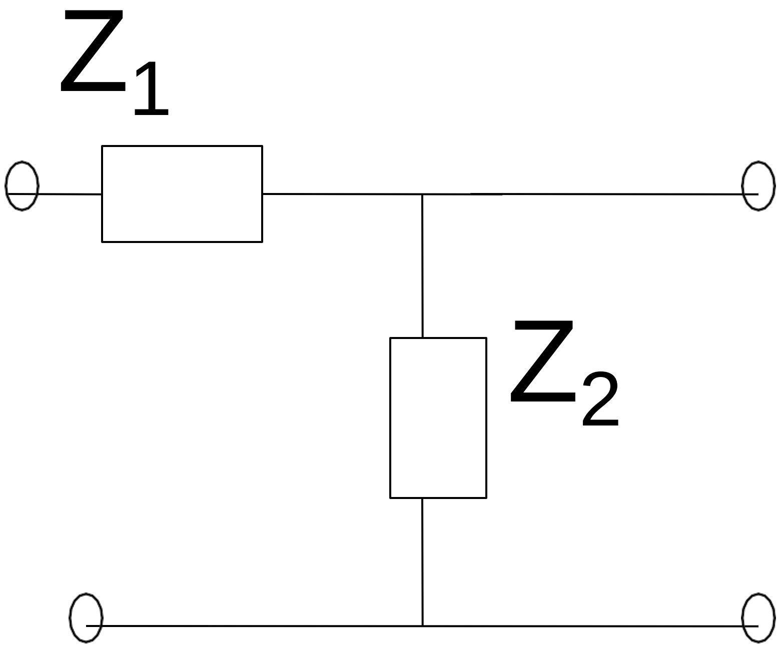




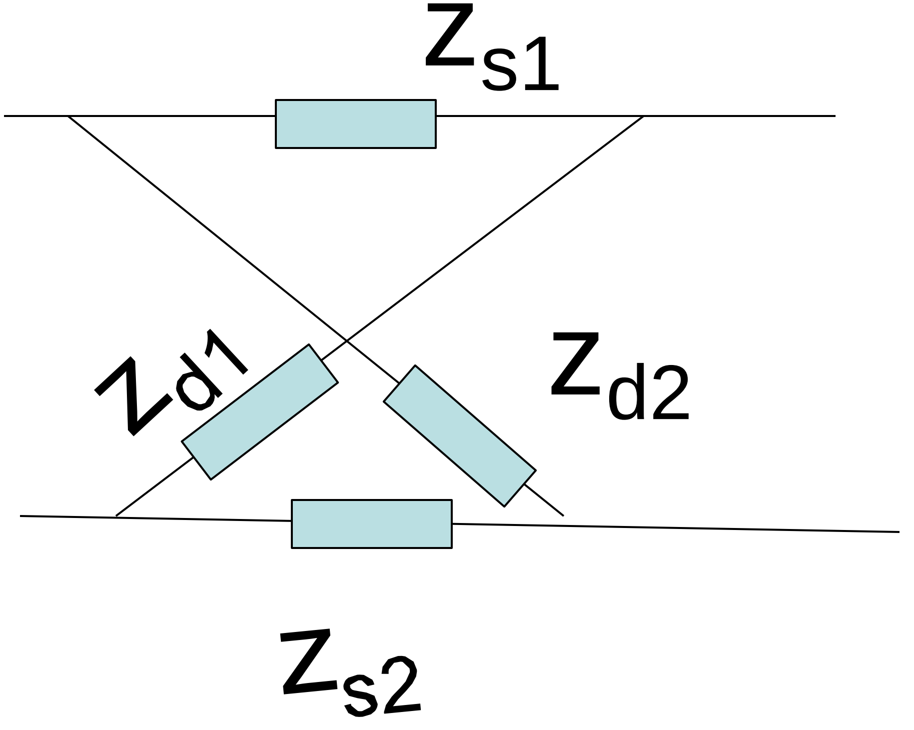
L-SECTION

When the network section looks like alphabet ‘L’ the configuration is termed as ‘L section’. When one of the series. Impedance is made equal to Zero of asymmetrical T-section and one of the shunt impedances of asymmetrical π section is made equal to infinity, the resultant network is called L-section.

L-SECTION



LATTICE SECTION



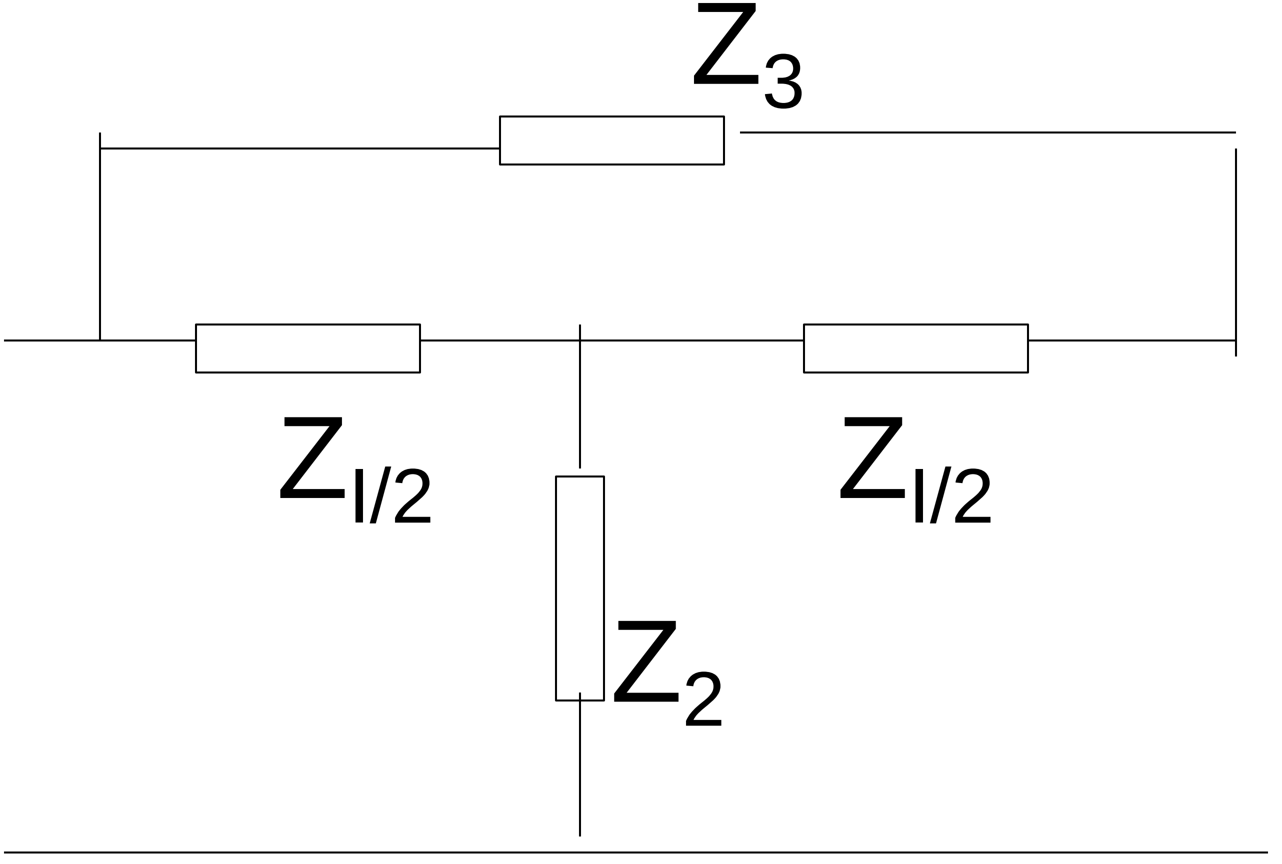
- Lattice section can be symmetrical and asymmetrical.

BRIDGE T-SECTION

- When an impedance used to bridge the series impedance of a symmetrical
- symmetrical and asymmetrical.

Network formed is called Bridge T network. Bridge T network can be

Bridge T-section



Symmetrical Network

Symmetrical Network have two important properties:

- (i) Characteristic Impedance 'Z_0'
- (ii) Propagation Constant 'γ'

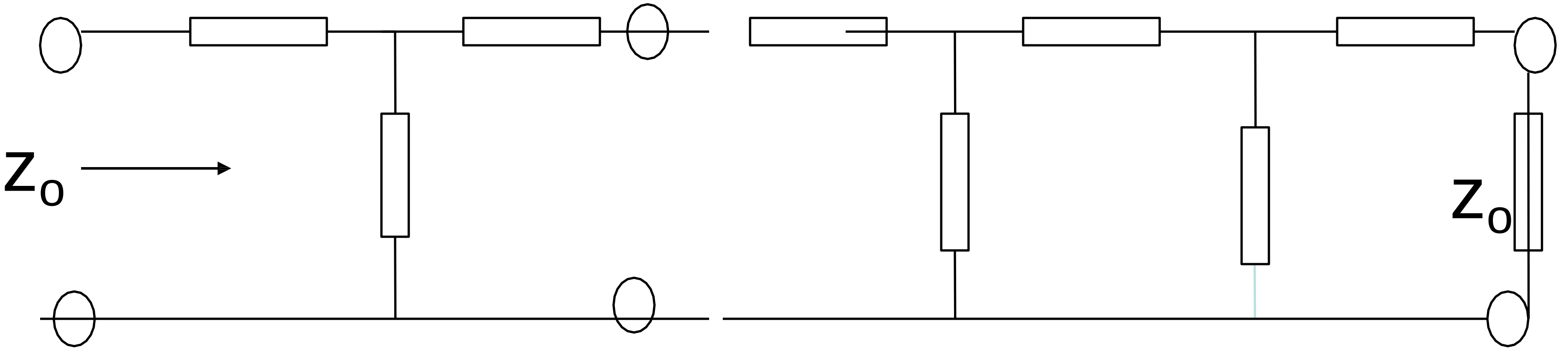
CHARACTERISTICS IMPEDANCE

- If an infinite number of identical symmetrical networks are joined. The impedance measured at the input end of the first network has a value which depends upon the network used. This impedance is the characteristics impedance $Z_{0.}$
- Characteristic Impedance value can be known if the component values of the network are known.

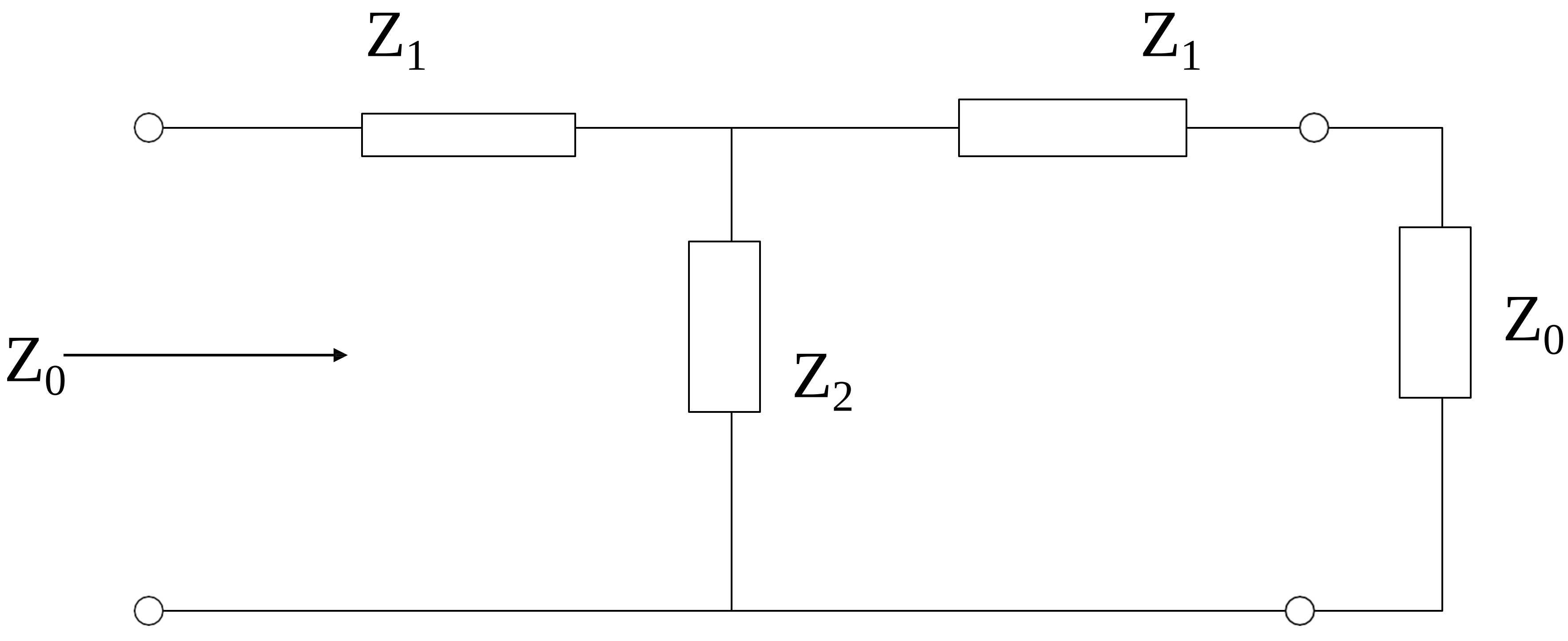
Characteristics Impedance

- It is the impedance which when connected at one port of symmetrical n/w produces same impedance at other port of n/w.
- If both input and output are terminated in Z_0, the network is said to be correctly or property terminated.

CHARACTERISTICS IMPEDANCE



CHARACTERISTICS IMPEDANCE



Evaluation of characteristic impedance

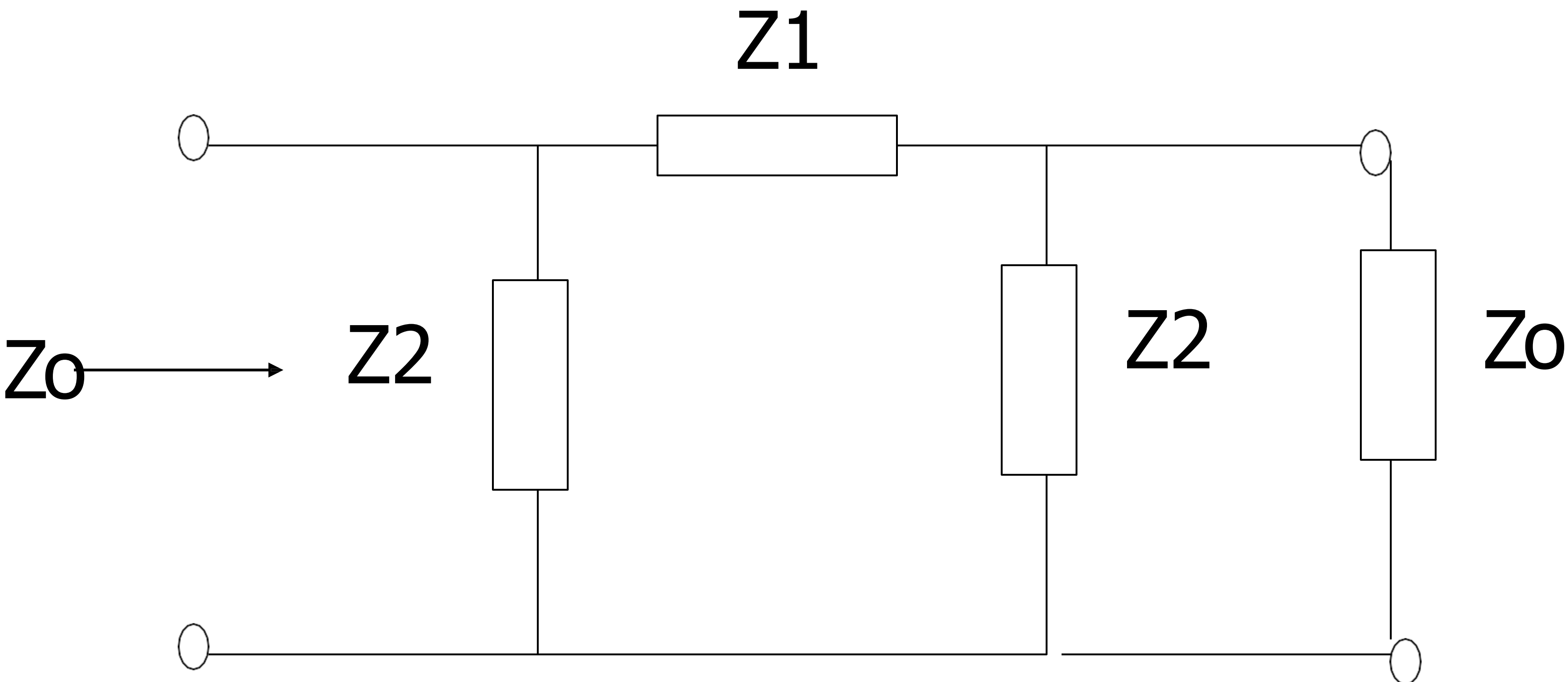
$$Z_{0T} = z_1/2 + \frac{[z_1/2 + z_0]z_2}{Z_1/2 + z_0 + z_2}$$

Solving this equation, We get

$$Z_{oT} = \sqrt{\frac{Z_1^{\,2}}{4} + Z_1 Z_2}$$

$$\mathbf{Z_{oT} = z_1 z_2 (1 + z_1/4z_2)}$$

SYMMETRICAL π NETWORK



$$Zo\ \pi = \sqrt{\frac{Z1Z2}{1+Z1/4Z2}}$$

Relation Between Z_{OT} & $Z_{O\pi}$

$$Z_{o\pi} = \frac{Z1Z2}{Z_{OT}}$$

Chacteristic impedeance is also given by:

$$\sqrt{Z_{OC}\, Z_{SC}}$$

Propagation Constant

- **Symmetrical networks have another property called propagation constant which gives relation between the input and output current or voltage.**
- **Propagation Constant is generally denotated by 'γ' ,which is a complex number.**

$$\& \ \gamma = \alpha + j\beta$$

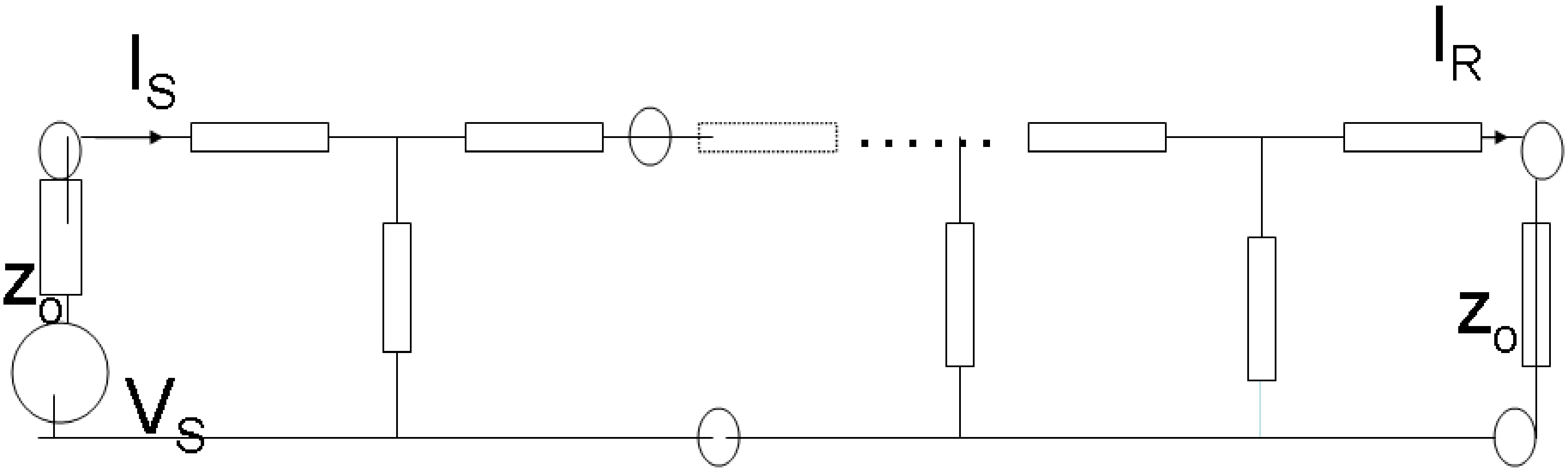
For a recurrent n/w,having cascaded identical symmetrical n/w sections:

$$\frac{I_S}{I_1} = \frac{I_1}{I_2} = \frac{I_2}{I_3} \text{......} = e^{\gamma}$$

For two sections: $$\frac{Is}{I_1} = \frac{I_1}{I_2} = e^{\gamma}$$

PROPAGATION CONSTANT

Symmetrical Network



And for n-section,

current ratio becomes

$$I_s / I_r = e^{n\gamma}$$

$$\& \quad \gamma = \alpha + j\beta$$

$$e^{\gamma} = e^{\alpha} + e^{j\beta} = e^{\alpha} . e^{j\beta}$$

$$e^{\gamma} = e^{\alpha} (\cos\beta + j\sin\beta)$$

$$= e^{\alpha} \sqrt{\cos^2\beta + \sin^2\beta} \; \angle \tan^{-1} \frac{\sin\beta}{\cos\beta}$$

$$= e^{\alpha} \angle \beta$$

Propagation Constant

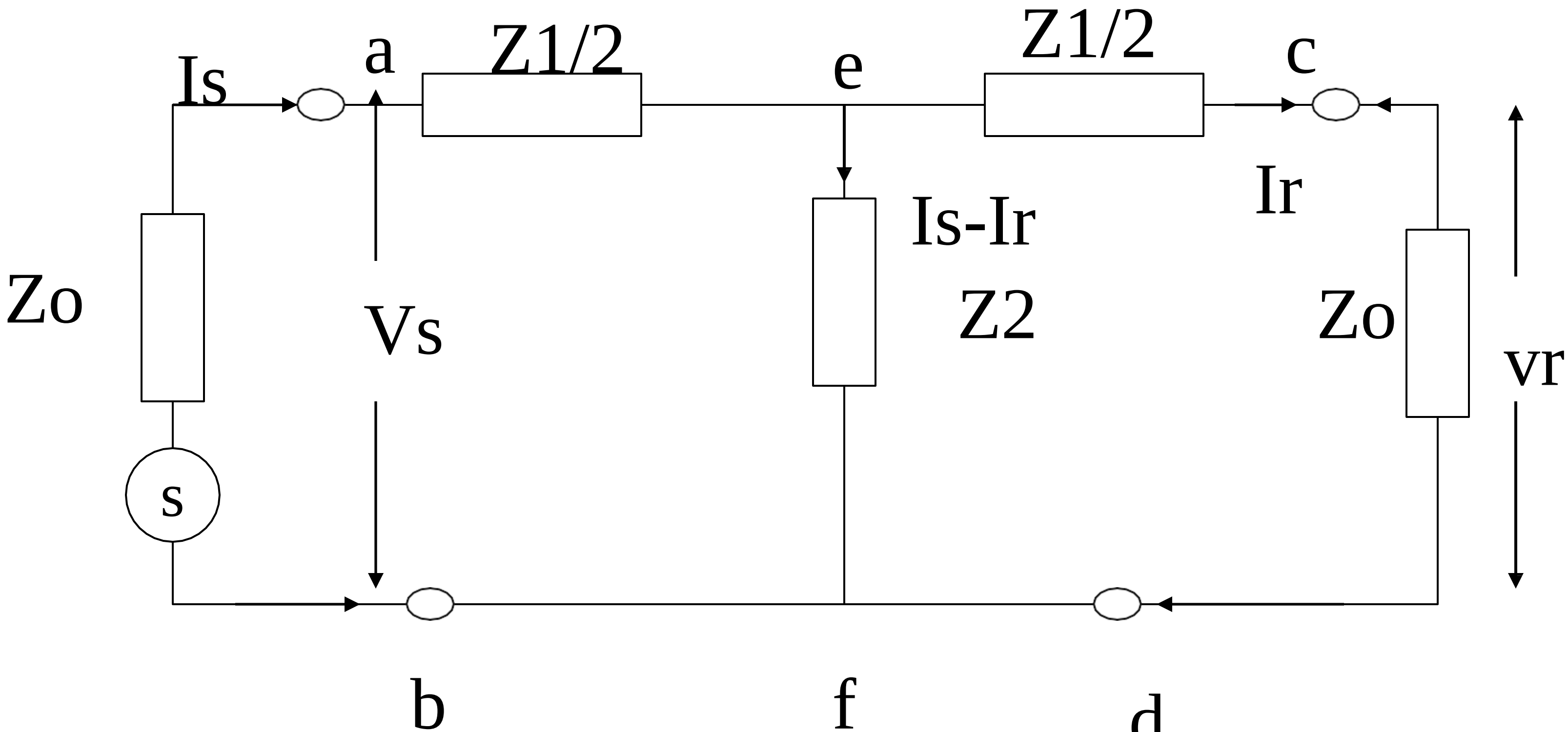
Attenuation constant(α):

It represents loss of signal energy during propagation through the network in terms of magnitude. Unit of α are **nepers.**

Phase shift constant(β):

It represents change of phase of the signal during its propagation through network. Unit of β are **radians.**

Propagation Constant of a Symmetrical T-network



$$\frac{Is}{Ir} = \frac{Z1}{2Z_2} + \frac{Zo}{Z2} + 1$$

$$\gamma = \mathbf{Cos\ h^{-1}[1+Z1/2Z2]}$$

ASYMMETRICAL NETWORK

Asymmetrical networks have different input and output impedance.

PROPERTIES OF ASYMMETRICAL NETWORKS

- ❖ Iterative impedance
- ❖ Image impedance
- ❖ Image Transfer Constant
- ❖ Iterative Transfer Constant

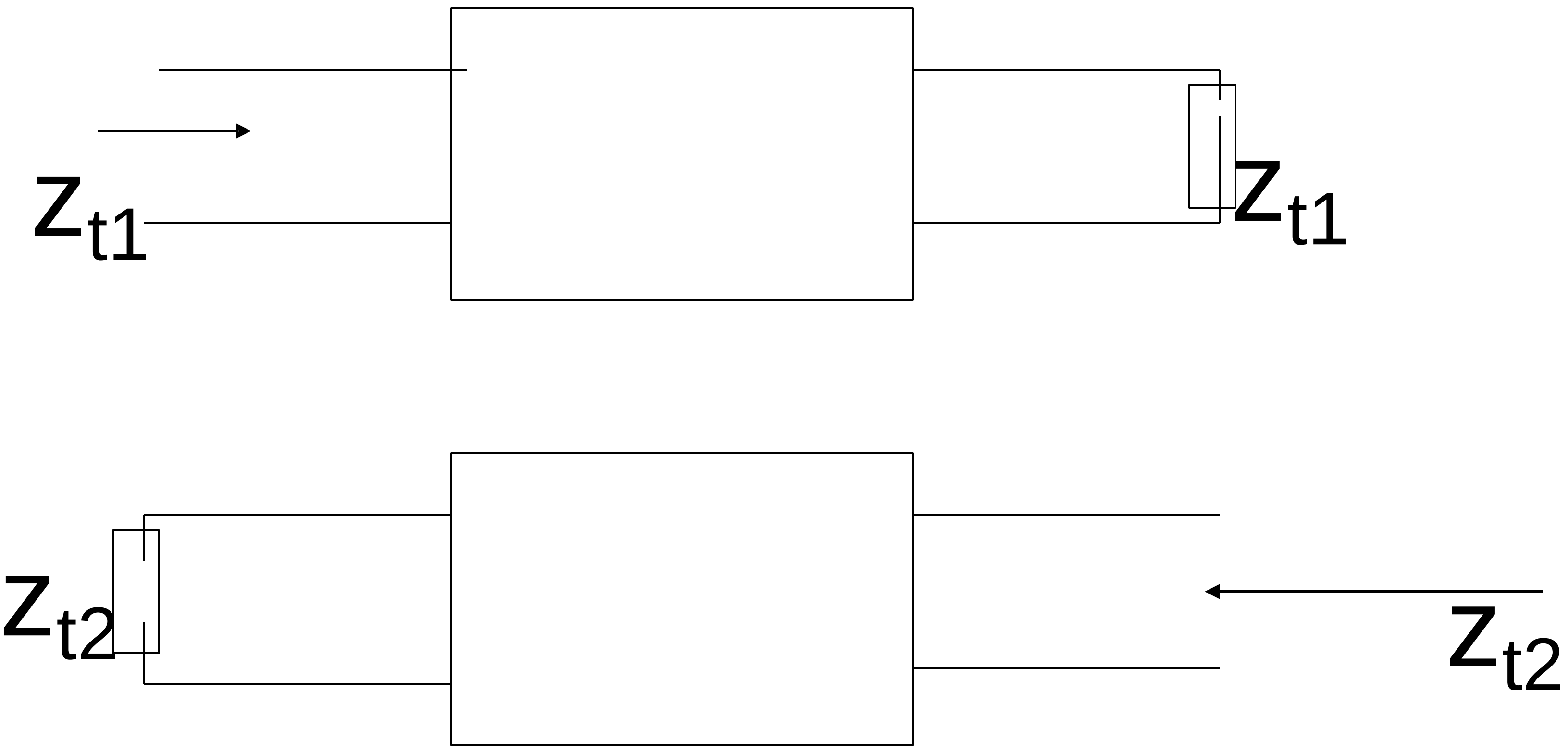
Iterative Impedance

- For a two port Asymmetrical network, iterative impedance is defined as the input impedance measured at one pair of terminals when other pair of terminals is terminated with an impedance of the same value.
- It may be defined as the impedance which will terminate the other terminals in such a way that the impedance measured at the first pair of terminals is equal to the terminating impedance.

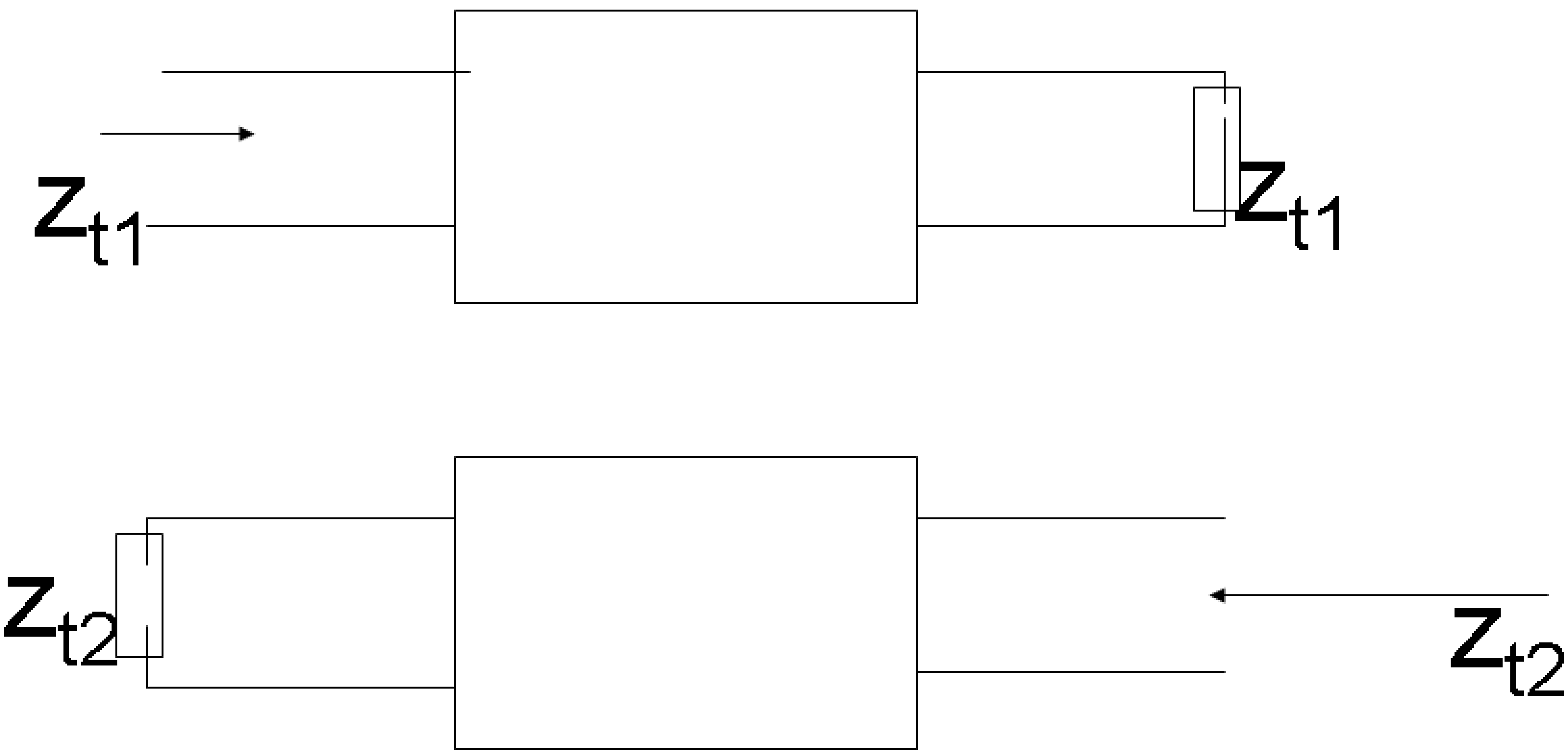
Iterative Impedance

- ✓ Iterative impedance are different for the two pair of terminals.
- ✓ Each asymmetrical network has two iterative, termed as Z_{t1} and Z_{t2}.

Iterative Impedance



Iterative Impedance



IMPEDANCE

- Image impedance of a network are those impedance, such that if one is connected across the proper pair of terminals of the network, the other is presented by the other pair of terminal.
- Every asymmetrical network has two different image impedances.
- Asymmetrical network is correctly terminated when it is terminated in its image impedances.

-Image impedances which simultaneously terminate each pair of terminals of a n/w in such a way that at each pair of terminals the impedance in both direction are equal.

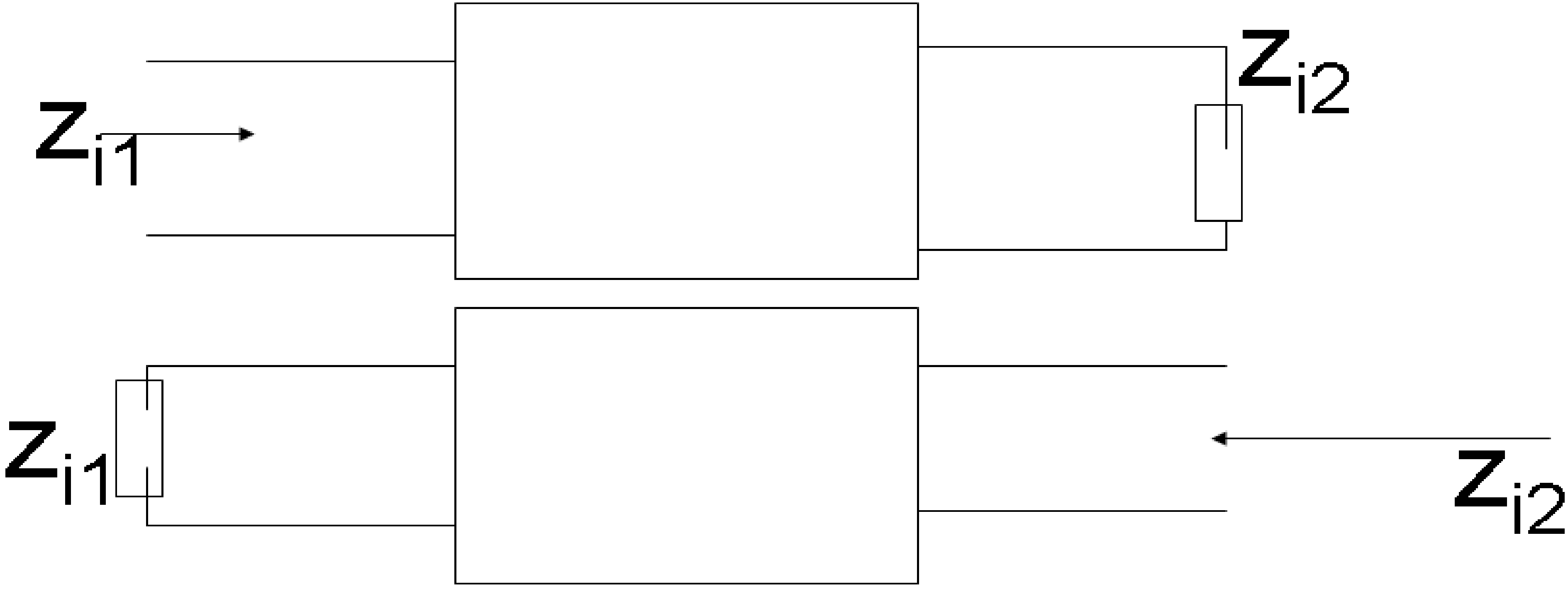


IMAGE TRANSFER CONSTANT 'Φ_I'

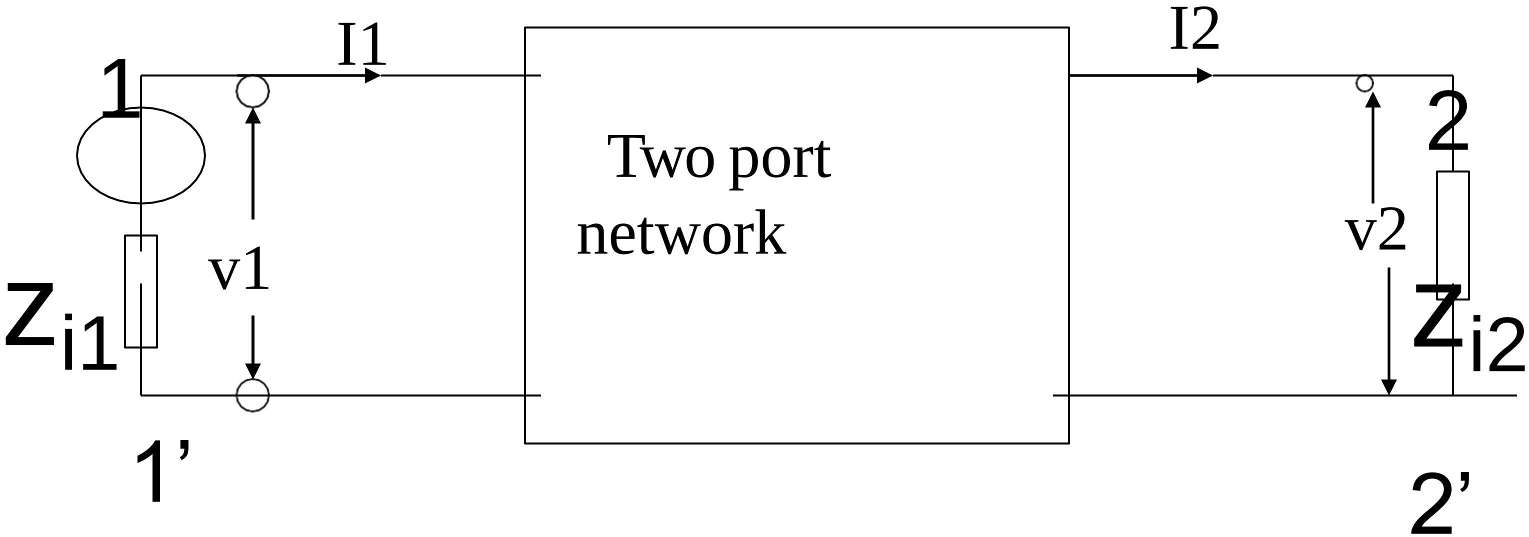
- In case of asymmetrical network, the factors affecting propagation of energy is defined in terms of **Image Transfer constant 'Φi'**, if the network terminated in image impedance.

- 'Φ_I' is also a complex quantity like 'γ' .

And $\Phi_I' = A_I + jB_I$

Here 'A_i' is a real term called Image attenuation constant. Units of A_i are nepers whereas 'B_i' is an imaginary term called Image phase shift constant. Unit of B_i are radians.

IMAGE TRANSFER CONSTANT 'ΦI'



$$e^{\Phi i} = \sqrt{\frac{V_1 I_1}{V_2 I_2}}$$

ITERATIVE TRANSFER CONSTANT 'Φ_T'

If asymmetrical network is terminated with iterative impedance, factors affecting the energy propagation are defined in terms of iterative constant 'Φ_t'. Iterative transfer constant is a complex number.

$$\Phi_t = A_t + jB_t$$

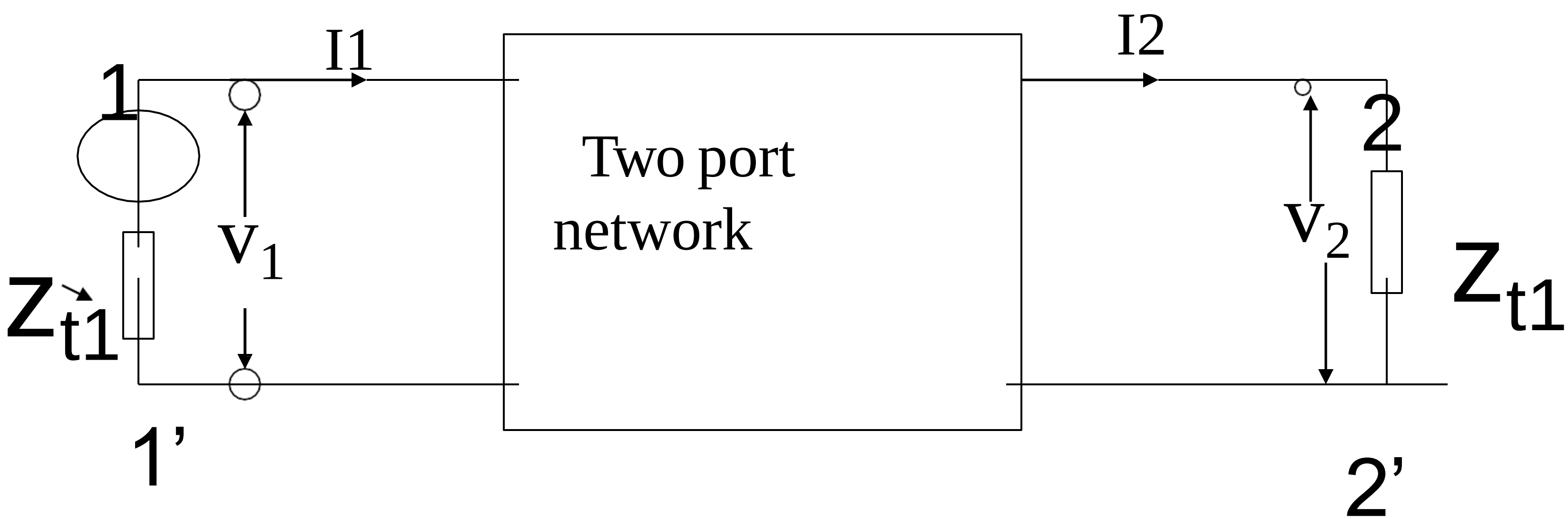
ITERATIVE TRANSFER CONSTANT 'Φ_T'

$$\Phi_t = A_t + jB_t$$

Where, A_t = _Iterative attenuation constant & unit of A_t are nepers.

B_t = Iterative phase constant & units of B_t are radians.

ITERATIVE TRANSFER CONSTANT 'ΦT'



$$e^{\Phi t} = v_1/v_2 = I_1/I_2$$

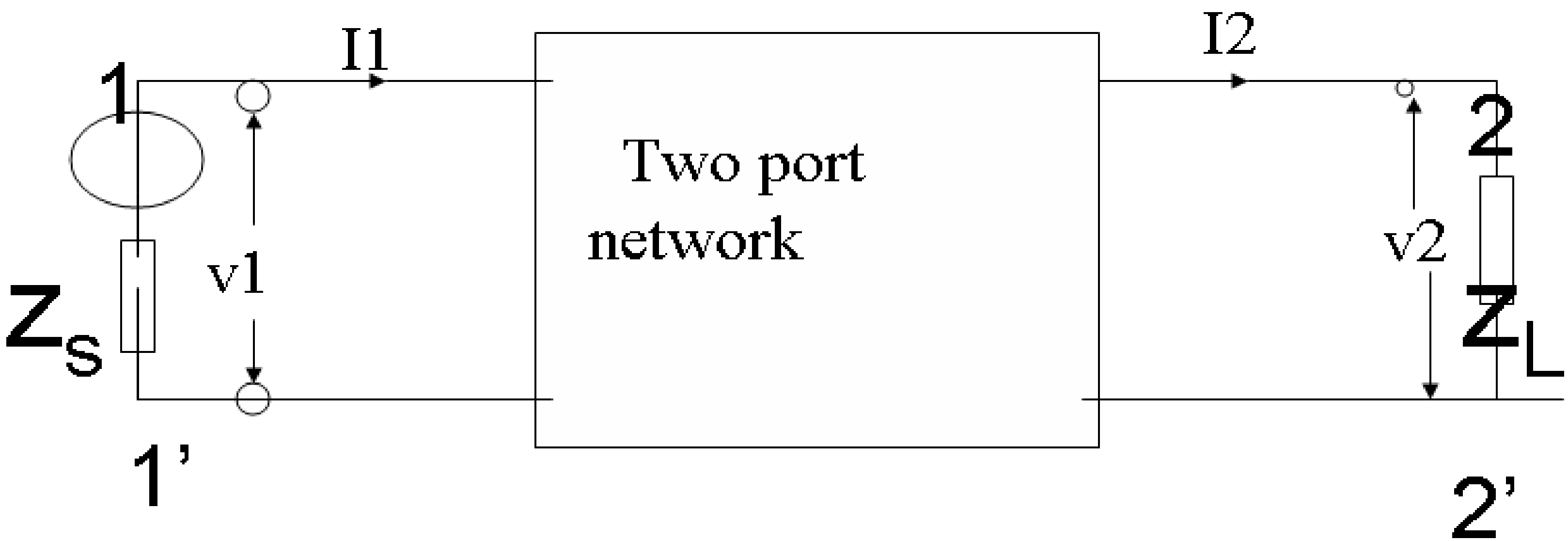
INSERTION LOSS

- The insertion loss of a network is defined as the number of nepers or decibels by which the signal power is reduced by the insertion.
- Whenever a passive network is inserted between an energy source and load, it generally produces a loss in received power.

INSERTION LOSS

- Insertion of a passive network introduces reduction in output voltage and current. This reduction in output voltage and current is termed as insertion loss and is measured in nepers (Np) and decibels (dB).
- Insertion loss not only depends upon the network but also depends upon source and load impedance.

INSERTION LOSS



I1 = Load current without insertion of N/W

I2 = Load current after insertion of N/W

Z_S = Source impedance Z_L = Load impedance

INSERTION LOSS

Ratio of I1/I2 is termed as the Insertion ratio of network.

So Insertion Ratio = I_1/I_2

Insertion loss =

$$20 \log_{10}\left|I_1/I_2\right| \text{db} = \log_e \left|I_1/I_2\right| \text{Np}$$

INSERTION LOSS

Insertion loss mainly depends upon;

- Attenuation constant & phase shift constant.
- Impedance matching between;

(a) Source & load impedance.

(b)Input impedance of the network & source impedance.

(c)Output impedance of the network & load impedance.

UNIT -II

ATTENUATOR

ATTENUATOR NETWORK:

- An attenuator network must fulfil following conditions.

- **It must give correct input impedance,**
- **It must give correct output impedance and**
- **It must provide specified attenuation.**

ATTENUATOR NETWORK:

In general, attenuation is expressed in decibel

$$D = 10 \log_{10} \left| \frac{P_{in}}{P_{out}} \right|$$

where D is the attenuation in decibel.

But we can express attenuation in neper as

$$D = 20 \log_{10} \sqrt{\frac{P_{in}}{P_{out}}} = 20 \log_{10} N$$

where N is the attenuation in neper.

$$N = \text{Antilog}_{10}\left(\frac{D}{20}\right) \quad \text{... (1)}$$

In this topic, we shall study symmetrical attenuators such as symmetrical T type, symmetrical π type, lattice type and bridged T type, along with asymmetrical attenuator such as ‘L’ type attenuator.

Any Attenuator Network is designed for specified characteristic resistance R_0 and attenuation.

Let us find design equations for various Attenuator Network one by one.

Symmetrical T Type Attenuator:

Consider properly terminated symmetrical T network as

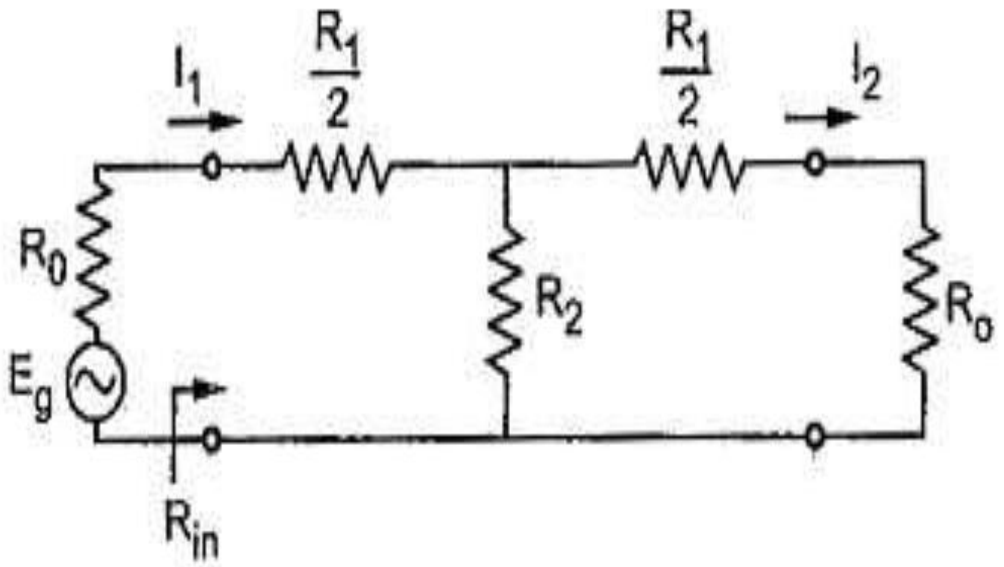


Fig. 1 Symmetrical T Type Attenuator

According to current divider rule,

$$I_2 = I_1 \left[\frac{R_2}{R_2 + \left(R_0 + \frac{R_1}{2} \right)} \right] \quad \text{... (1)}$$

But for symmetrical networks,

$$N = \frac{I_1}{I_2} = \frac{R_0 + R_2 + \frac{R_1}{2}}{R_2} \quad \text{... (2)}$$

For properly terminated network, input impedance R_{in} is give

$$R_{in} = R_0 = \left[\left(R_0 + \frac{R_1}{2} \right) \parallel R_2 \right] + \frac{R_1}{2}$$

$$R_0 = \frac{R_2 \left(R_0 + \frac{R_1}{2} \right)}{R_0 + R_2 + \frac{R_1}{2}} + \frac{R_1}{2}$$

From equation (2),

substituting $\frac{1}{N}$ for $\frac{R_2}{R_0 + R_2 + \frac{R_1}{2}}$

$$R_0 = \frac{R_0 + \frac{R_1}{2}}{N} + \frac{R_1}{2}$$

$$N R_0 = R_0 + \frac{R_1}{2} + N\frac{R_1}{2}$$

$$R_0(N-1) = \frac{R_1}{2}(N+1)$$

$$\frac{R_1}{2} = R_0\left(\frac{N-1}{N+1}\right) \qquad \text{... (A)}$$

From equation (2), we can write,

$$N R_2 = R_0 + R_2 + \frac{R_1}{2}$$

$$R_2(N-1) = R_0 + R_0\left(\frac{N-1}{N+1}\right)$$

$$R_2\left(N^2-1\right) = R_0(N+1) + R_0(N-1)$$

$$R_2 = R_0\left(\frac{2N}{N^2-1}\right) \qquad \text{... (B)}$$

Equations (A) and (B) are called design equations of symmetrical T attenuator.

Symmetrical π Type Attenuator:

Consider properly terminated symmetrical π network as shown i

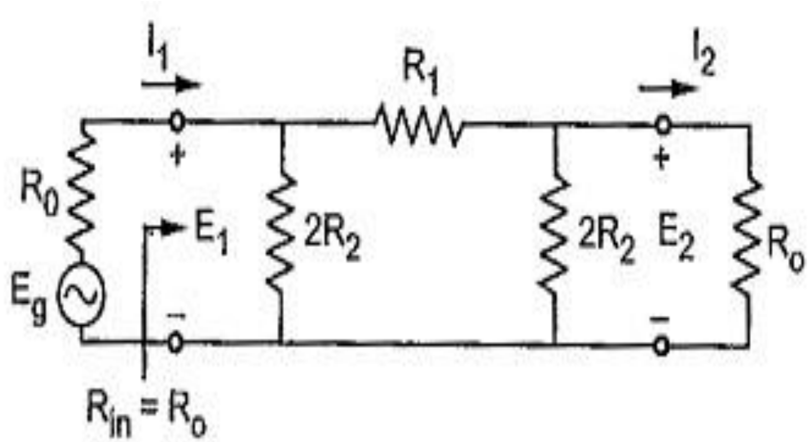


Fig - 2 **Symmetrical π Type Attenuator**

Let the characteristic impedance be pure resist
propagation constant γ = α. Then according to t
symmetrical network, the shunt arm and series
expressed in terms of R_0 and α as follows,

$$R_1 = R_0 \sinh \alpha \quad \text{... (1)}$$

$$2R_2 = \frac{R_0}{\tanh \frac{\alpha}{2}} \quad \text{... (2)}$$

$$R_1 = R_0 \left[\frac{e^{\alpha} - e^{-\alpha}}{2}\right]$$

$$N = \frac{I_1}{I_2} = e^{\alpha}$$

Simplifying equation (1)…

$$R_1 = R_0\left[\frac{N-\frac{1}{N}}{2}\right] = \frac{R_0}{2}\left[\frac{N^2-1}{N}\right]$$

$$R_1 = R_0\left[\frac{N^2-1}{2N}\right] \qquad \text{... (A)}$$

$$2R_2 = R_0\left[\frac{1}{\tanh e^{\alpha/2}}\right] = R_0\left[\frac{1}{\frac{e^{\alpha/2}-e^{-\alpha/2}}{e^{\alpha/2}+e^{-\alpha/2}}}\right]$$

$$2R_2 = R_0\left[\frac{e^{\alpha/2}+e^{-\alpha/2}}{e^{\alpha/2}-e^{-\alpha/2}}\right]$$

Simplifying equation (2),

Multiplying numerator and denominator by factor $e^{\alpha/2}$ on right hand side

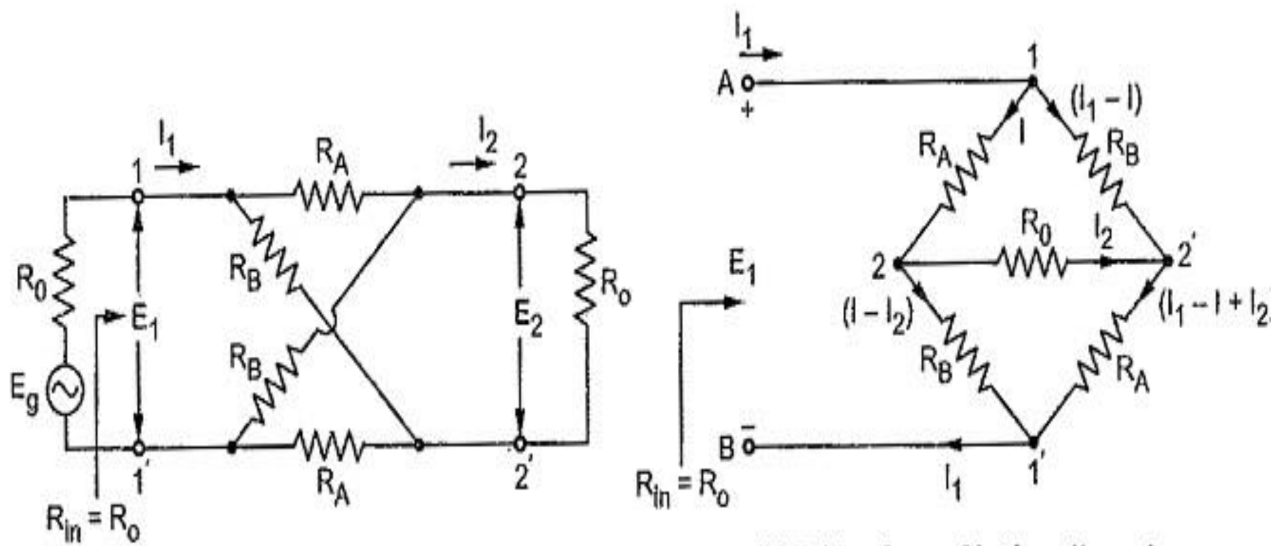
$$2R_2 = R_0\left[\frac{e^{\alpha}+1}{e^{\alpha}-1}\right]$$

$$2R_2 = R_0\left[\frac{N+1}{N-1}\right] \qquad \text{... (B)}$$

Equations (A) and (B) are called design equations of symmetrical π attenuator.

Symmetrical Lattice Type Attenuator:

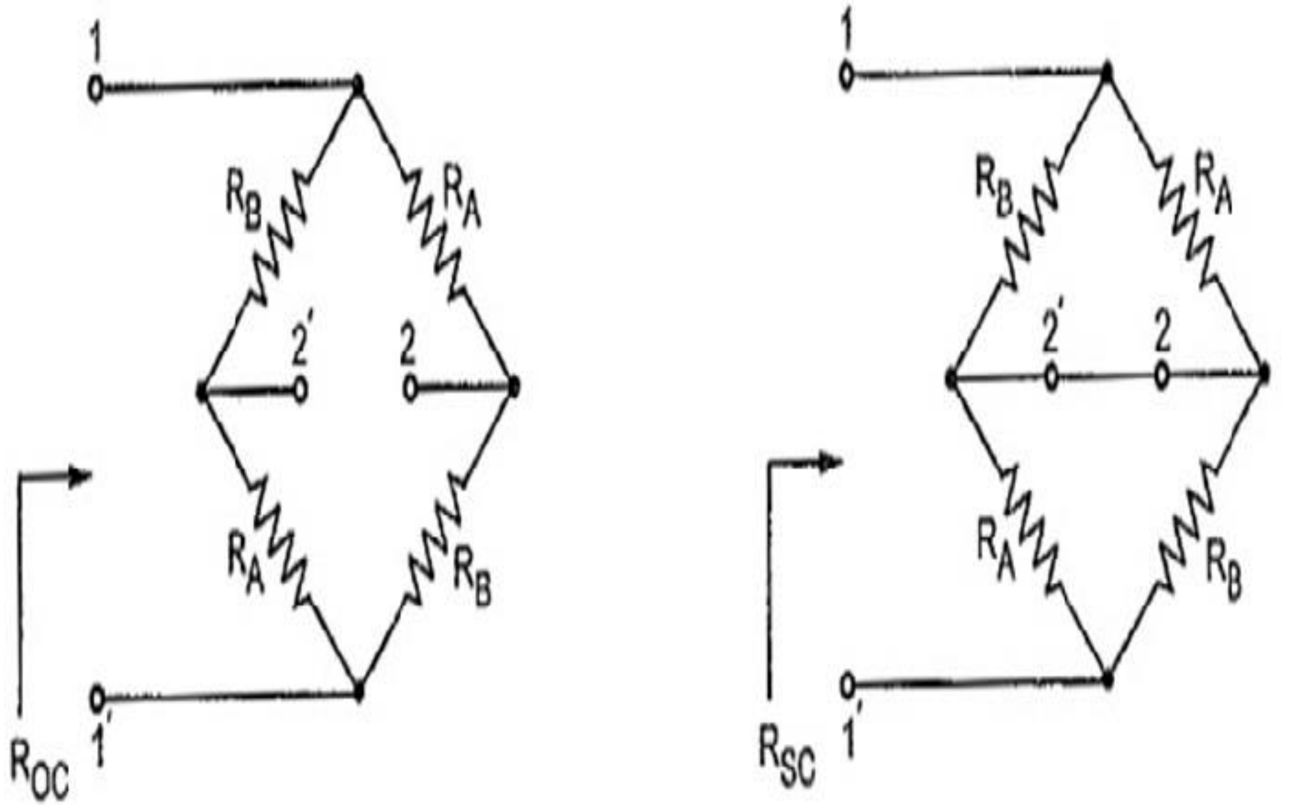
Consider properly terminated lattice Attenuator Network as shown in the F



(a) Lattice attenuator (b) Bridge form of lattice attenuator

Fig. 3 Symmetrical Lattice Type Attenuator

According to the theory of the symmetrical networks, characteristic impedance is the geometric mean of open and short cir



(a) Output terminals open circuited

(b) Output terminals short circuited

Consider lattice network shown in the Fig. 3 (a) and (b) with open and short circuit output terminals respectively.

$$R_{OC} = (R_A + R_B) \parallel (R_A + R_B) = \frac{R_A + R_B}{2}$$

$$R_{SC} = [R_A \parallel R_B] + [R_A \parallel R_B] = 2\frac{R_A R_B}{R_A + R_B}$$

By definition, characteristic impedance is given by,

$$R_0 = \sqrt{R_{OC} R_{SC}} = \sqrt{R_A R_B} \qquad \dots (1)$$

Consider Fig. 3, applying KVL to closed path A-1-2-2′-1′-2-A, we can write,

$$-(R_A) I - R_0 (I_2) - R_A (I_1 - I + I_2) + I_1 R_0 = 0$$

$$\therefore \quad -R_A I - R_0 I_2 - R_A I_1 + R_A I - R_A I_2 = -R_0 I_1$$

$$(R_0 - R_A) I_1 = I_2 (R_0 + R_A) \qquad \dots (2)$$

Thus, we can write,

$$\frac{I_1}{I_2} = N = \frac{R_0 + R_A}{R_0 - R_A}$$

$$N(R_0 - R_A) = R_0 + R_A$$

$$R_A(N+1) = R_0(N-1)$$

$$R_A = R_0\left[\frac{N-1}{N+1}\right] \qquad \text{... (A)}$$

Applying KVL to closed path A-1-2′-2–1′-B-A, we can write,

$$-(R_B)(I_1 - I) + (R_0) I_2 - R_B(I - I_2) + I_1 R_0 = 0$$

$$-R_B I_1 + R_B I + R_0 I_2 - R_B I + R_B I_2 + R_0 I_1 = 0$$

$$I_2(R_0 + R_B) = I_1(R_B - R_0) \qquad \text{... (3)}$$

$$N = \frac{I_1}{I_2} = \frac{R_B + R_0}{R_B - R_0}$$

$$N(R_B - R_0) = R_B + R_0$$

$$R_B(N-1) = R_0(N+1)$$

$$R_B = R_0\left[\frac{N+1}{N-1}\right] \qquad \text{... (B)}$$

Equations (A) and (B) are called design equations of symmetrical lattice att

Bridged T Attenuator:

Consider properly terminated bridged T network as shown in the Fig. 4. Assuming 3 loop currents in clockwise direction as shown.

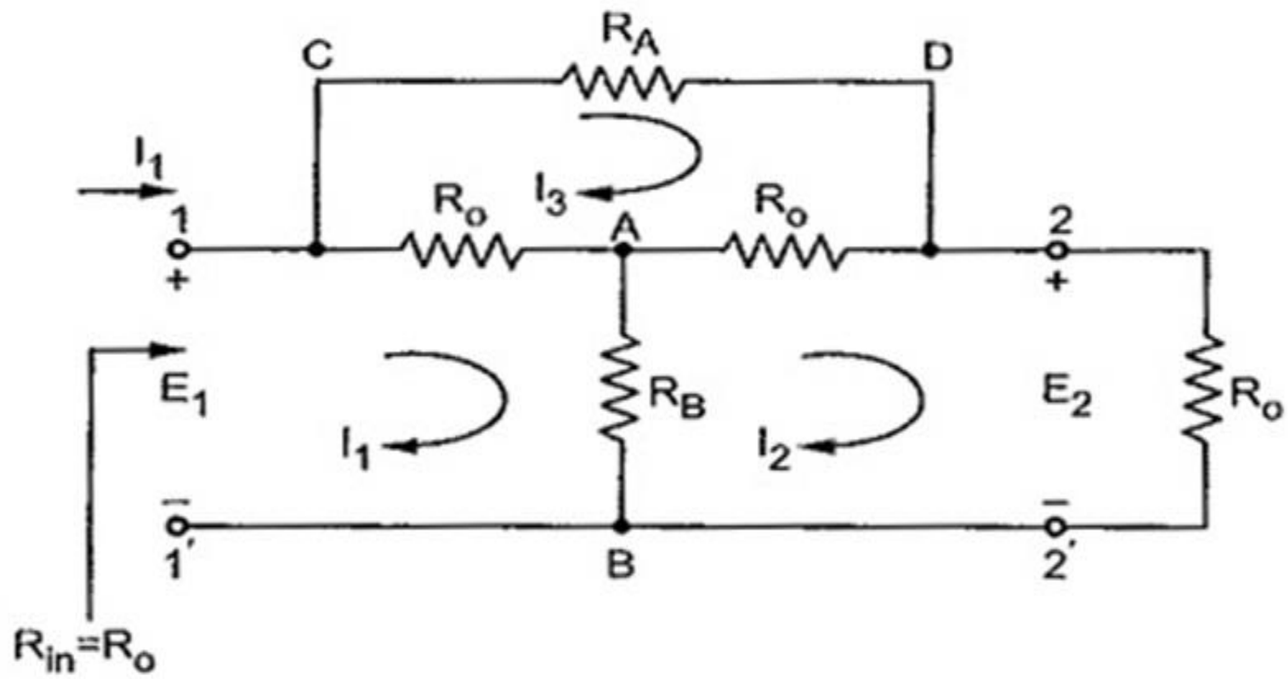


Fig – 4 Bridged T Attenuator

Consider closed path 1-A-B-1′-1, applying KVL,

$$-R_0\,I_1 + R_0\,I_3 - R_B\,I_1 + R_B\,I_2 + E_1 = 0$$

$$\therefore \quad -(R_0 + R_B)\,I_1 + R_B\,I_2 + R_0\,I_3 = -E_1$$

$$(R_0 + R_B)\,I_1 - R_B\,I_2 - R_0\,I_3 = -E_1$$

$$\therefore \quad (R_0 + R_B)\,I_1 - R_B\,I_2 - R_0\,I_3 = R_0 I_1 \;\ldots\ldots \text{ as } E_1 = R_o \cdot I_1$$

$$\therefore \quad R_B\,I_1 - R_B\,I_2 - R_0\,I_3 = 0 \qquad \ldots (1)$$

Consider closed path A-2-2′-B-A, applying KVL,

$$-R_0 I_2 - R_0 I_2 - R_B I_2 + R_B I_1 + R_0 I_3 = 0$$

$$\therefore \quad -(2\,R_0 + R_B)\,I_2 + R_B I_1 + R_0 I_3 = 0 \qquad \ldots (2)$$

Consider closed path C-D-2-A-1-C, applying KVL,

$$-R_A I_3 - R_0 I_3 + R_0 I_2 - R_0 I_3 + R_0 I_1 = 0$$

$$\therefore \quad R_0\,I_1 + R_0\,I_2 - (2\,R_0 + R_A)\,I_3 = 0 \qquad \ldots (3)$$

Adding equations (1) and (2),

$$2\,R_B\,I_1 - (2\,R_0 + 2\,R_B)\,I_2 = 0$$

$$(2\,R_B)\,I_1 = 2\,(R_0 + R_B)\,I_2$$

$$\frac{I_1}{I_2} = N = 1 + \frac{R_0}{R_B}$$

$$R_B = \frac{R_0}{N - 1} \qquad \ldots (A)$$

From equation (1) we can write,

$$R_B(I_1 - I_2) = R_0 I_3$$

$$I_3 = R_B\left[\frac{(I_1 - I_2)}{R_0}\right] \qquad \text{... (4)}$$

Substituting value of I_3 in

$$R_0 + R_A)\left[\frac{R_B(I_1 - I_2)}{R_0}\right]$$

Substituting value of R_B from equation (A) we can write,

$$R_0 I_1 + R_0 I_2 = (2R_0 + R_A)\left[\frac{R_0}{(N-1)}\frac{(I_T - I_2)}{R_0}\right]$$

$$[R_0 I_1 + R_0 I_2][N-1] = (2R_0 + R_A)(I_1 - I_2)$$

$$N R_0 I_1 + N R_0 I_2 - R_0 I_1 - R_0 I_2 = 2R_0 I_1 + R_A I_1 - 2R_0 I_2 - R_A I_2$$

$$I_1 [N R_0 - R_0 - 2R_0 - R_A] = I_2 [-2R_0 - R_A - N R_0 + R_0]$$

$$N = \frac{I_1}{I_2} = \frac{-(N R_0 + R_A + R_0)}{N R_0 - 3R_0 - R_A}$$

$$N^2 R_0 - 3N R_0 - N R_A = -N R_0 - R_A - R_0$$

$$-N R_A + R_A = -N^2 R_0 + 2N R_0 - R_0$$

$$-R_A(N-1) = -R_0\left(N^2 - 2N + 1\right)$$

$$R_A = \frac{R_0(N-1)^2}{(N-1)}$$

$$R_A = R_0(N-1) \qquad \text{... (B)}$$

Equations (A) and (B) are called design equations of bridged T attenuator.

L Type Asymmetrical Attenuator:

An asymmetrical L type attenuator is as shown in the Fig. 5.

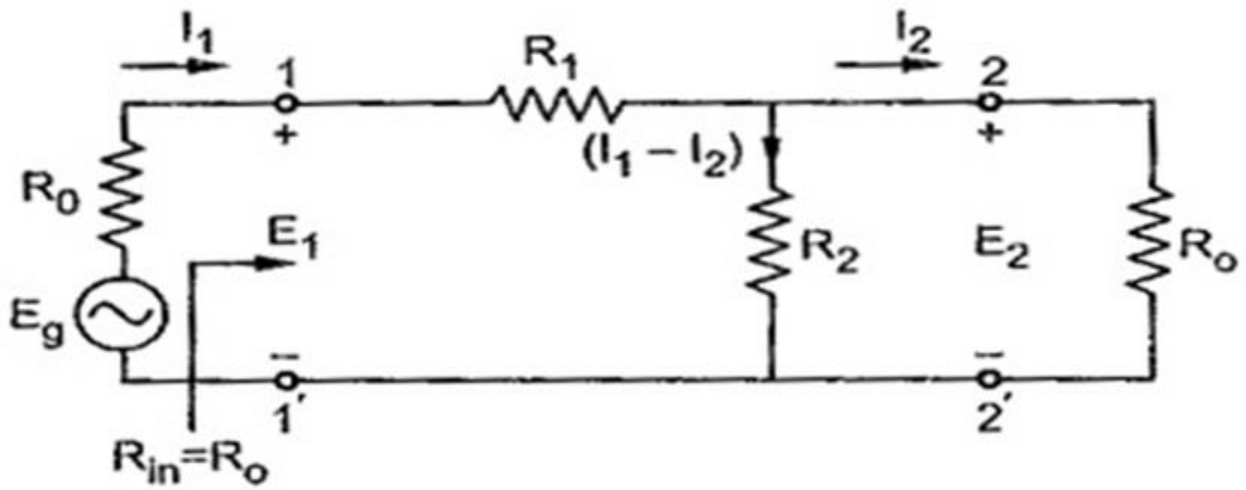


Fig – 5 L- Type Asymmetrical Attenuator

$$E_2 = (I_1 - I_2) R_2 = I_2 R_0$$

$$I_1 R_2 = I_2 (R_0 + R_2)$$

$$\frac{I_1}{I_2} = N = \frac{R_0 + R_2}{R_2}$$

$$N R_2 = R_0 + R_2$$

$$R_2 = R_0 \left[\frac{1}{N-1}\right] \qquad \text{... (A)}$$

Input resistance looking into network from terminals 1-1′ is

$$R_{in} = R_0 = R_1 + \frac{R_2 R_0}{R_2 + R_0}$$

$$R_0 (R_2 + R_0) = R_1 (R_2 + R_0) + R_2 R_0$$

$$R_2 R_0 + R_0^2 = R_1 R_2 + R_1 R_0 + R_2 R_0$$

Putting value of R_2 from equation (A),

$$R_0^2 = R_1\left[\frac{R_0}{N-1}\right] + R_1 R_0$$

$$R_0 = \frac{R_1}{N-1} + R_1$$

$$R_0(N-1) = R_1 + N R_1 - R_1$$

$$R_1 = R_0\left[\frac{N-1}{N}\right] \quad \dots (B)$$

equations (A) and (B) are called design equations of asymmetrical L type attenuator.

CHAPTER-3

FILTERS

Low Pass Filter:

The prototype T and π low pass filter sections are as shown in the Fig. 1.

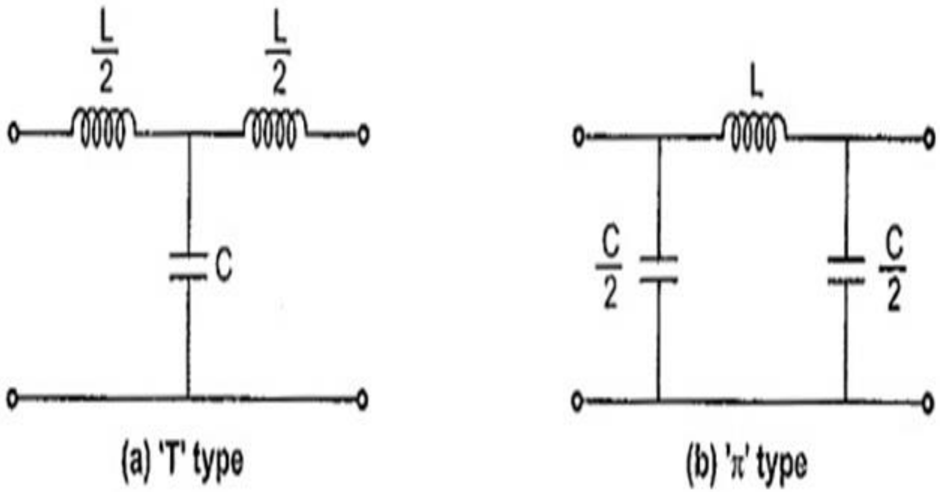


Fig 1 Prototype T and π low pass filter

Design Impedance (R_0):

Here in low pass filter sections,

Total series arm impedance Z_1 = jωL

Total shunt arm impedance Z_2 = -j/ωC

Hence, $Z_1 . Z_2$ = (jωL) (-j/ωC) = L/C which is real and constant. Hence sections are constant K sections so we can write,

$$R_0^2 = Z_1 \cdot Z_2 = \frac{L}{C}$$

$$R_0 = \sqrt{\frac{L}{C}} \qquad \dots (1)$$

Reactance Curves and Cut-off Frequency Expression:

As both T and π sections have same cut-off frequency, it is sufficient to calculate f_c for the 'T' section only.

$$Z_1 = (j\omega L) \qquad \therefore \quad X_1 = \omega L$$

$$Z_2 = \frac{-j}{\omega C} \qquad \therefore \quad X_2 = \frac{-1}{\omega C}$$

$$\frac{X_1}{4} + X_2 = \frac{\omega L}{4} - \frac{1}{\omega C}$$

The reactance curves are as shown in the Fig. 2

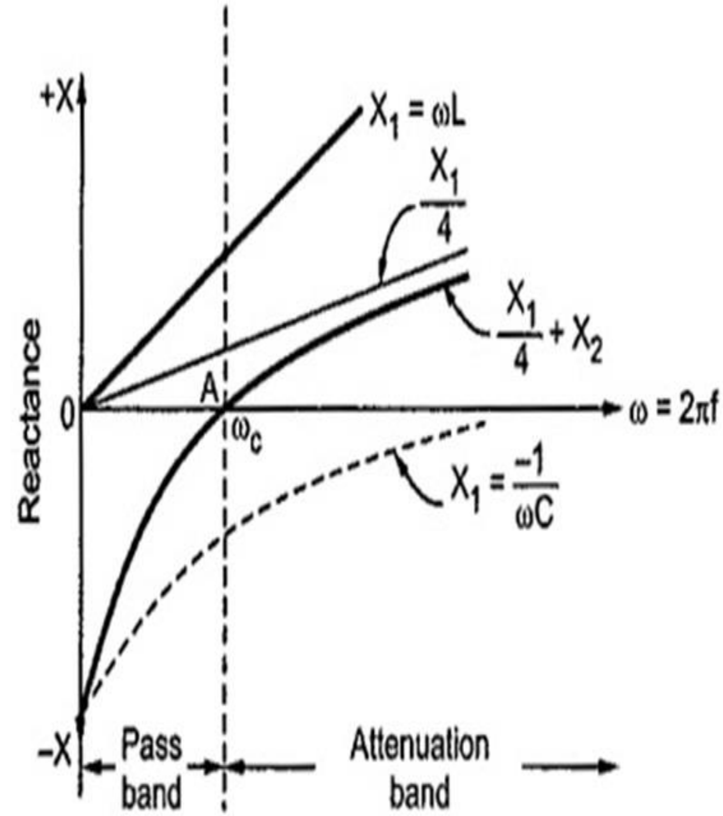


Fig 2 Reactance Frequency sketch for prototype low pass filter

From above characteristic it is clear that all the reactance curves have positive slope as a curves slope upward to the right side with increasing ω.

The curves are on opposite sides of the frequency axis upto point A; while on the same side, from point A on wards. Hence all the frequencies upto point A give pass band and above point A give stop band. Thus point A marks cut-off frequency given by $\omega = \omega_c$.

At point A, $\omega = \omega_c$, the curve for $(X_1/4 + X_2)$ crosses the frequency axis, hence we can write,

$$\frac{\omega_c L}{4} - \frac{1}{\omega_c C} = 0$$

$$\frac{\omega_c L}{4} = \frac{1}{\omega_c C}$$

$$\omega_c^2 = \frac{4}{LC} \qquad \dots (2)$$

$$\omega_c = \frac{2}{\sqrt{LC}} \quad \text{or}$$

$$f_c = \frac{1}{\pi\sqrt{LC}} \qquad \dots (3)$$

The algebraic approach to calculate cut-off frequency is as follows

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$= \sqrt{\frac{-\omega^2 L^2}{4} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}} \cdot \sqrt{1 - \frac{\omega^2 LC}{4}}$$

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}} \qquad \ldots (4)$$

From above expression it is clear that, Z_{0T} is real if $\omega^2 LC/4 < 1$ and imaginary if $\omega^2 LC/4 >$
Hence condition $\omega^2 LC/4 - 1 = 0$ gives expression,

$$\omega^2 = \frac{4}{LC} \quad \text{or} \quad \omega = \frac{2}{\sqrt{LC}}$$

Thus, above prototype section passes all frequencies below $\omega = 2/\sqrt{LC}$ while attenuates a
this value. Therefore cut-off frequency of low pass filter is given by

$$\omega_c = \frac{2}{\sqrt{LC}} \quad \text{or} \quad f_c = \frac{1}{\pi\sqrt{LC}}$$

Above frequency comes out to be the same
as calculated by reactance sketch method.

Variation of Z_{0T} and $Z_{0\pi}$ with Frequency:

Consider expression

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}}$$

From equation (2), we can write

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2}{\omega_c^2}} \quad \text{or}$$

$$Z_{0T} = R_0 \sqrt{1 - \left(\frac{f}{f_c}\right)^2} \qquad \dots (5)$$

Similarly we can write,

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

$$Z_{0\pi} = \frac{R_0^2}{R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

Hence

$$Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} \qquad \ldots (6)$$

From equation (5), it is clear that as frequency increases from 0 to f_c, Z_{0T} decreases from
For π section, from equation (6),
it is clear that in pass band as frequency increases for 0 to f_c, $Z_{0\pi}$ increases from R_0 to ∞
The variation of Z_{0T} and $Z_{0\pi}$ with frequency is as shown in the Fig. 3

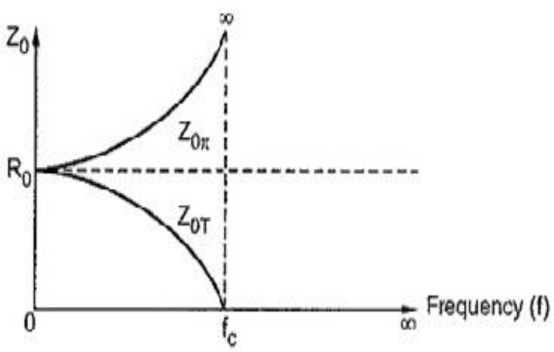


Fig 3 Variation of characteristics impedance with frequency

Variation of Attenuation Constant α with Frequency:

In pass band attenuation is zero. In stop band attenuation is given by,

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right) \qquad \dots (7)$$

band as frequency f increases above f_c, attenuation also increases. The variation of α with frequency is as shown in the Fig. 4.

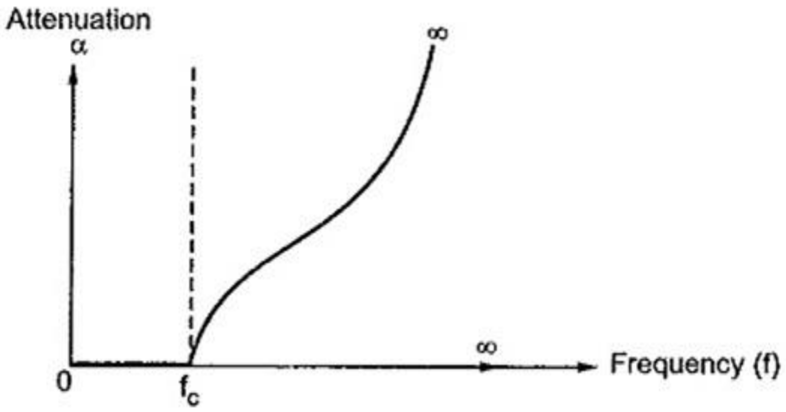


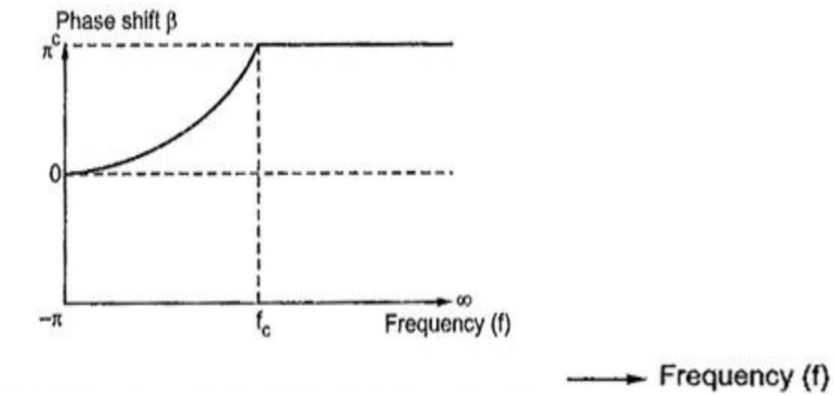
Fig 4 Variations of attenuation constant with frequency

Variation of Phase Constant β with Frequency:

In stop band, phase constant β is always equal to π radian. In pass band where α = 0, the phase constant β is given by

$$\beta = 2\sin^{-1}\left(\frac{f}{f_c}\right) \qquad \dots (8)$$

As frequency increases from 0 to f_c, β also increases from 0 to π radian. The variation of β



Variation of phase constant with frequency

Design Equations of Prototype Low Pass Filter:

The design impedance R_0 and cut-off frequency f_c can be given in terms of L and C as follows.

$$R_0 = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

Dividing equation for R_0 by f_c, we get,

$$L = \frac{R_0}{(\pi f_c)} \quad \ldots (9)$$

Multiplying equation for R_0 and f_c we get,

$$C = \frac{1}{(\pi f_c) R_0} \quad \ldots (10)$$

Equations (9) and (10) are called design equations for prototype low pass filter sections.

High Pass Filter:

The prototype high pass filter T and π sections are as shown in the Fig. 6.

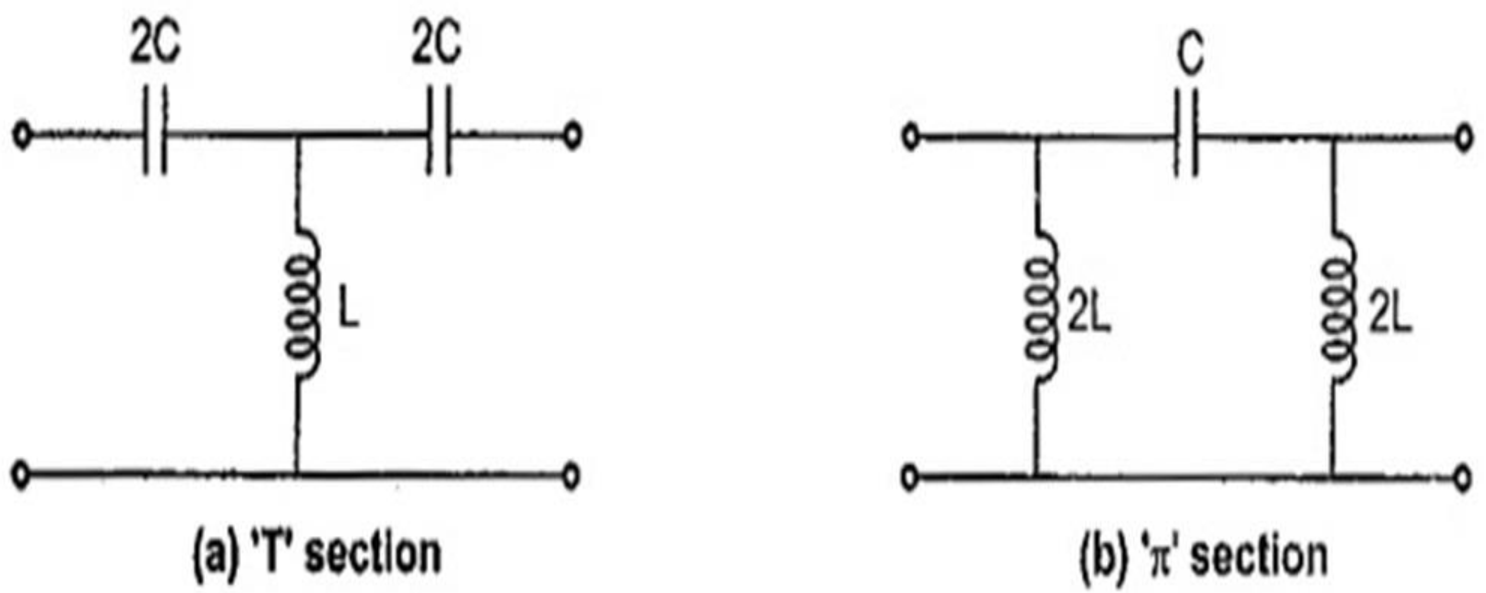


Fig 6 Prototype T and π high pass filter

Design Impedance (R_0):
Total series arm impedance Z_1 = -j/ωC
Total shunt arm impedance Z_2 = jωL
Hence, $Z_1 . Z_2$ =(-j/ωC) (jωL) = L/C which is real and constant. Hence above sections are constant K sections. So we can write,

$$R_0^2 = Z_1 \cdot Z_2 = \frac{L}{C}$$

$$R_0 = \sqrt{\frac{L}{C}} \qquad \ldots (1)$$

$$Z_1 = \frac{-j}{\omega C} \qquad \therefore \quad X_1 = \frac{-1}{\omega C}$$

$$Z_2 = (j\omega L) \qquad \therefore \quad X_2 = \omega L$$

$$\frac{X_1}{4} + X_2 = \frac{-1}{4\omega C} + \omega L = \omega L - \frac{1}{4\omega C}$$

Reactance Curves and Expression for Cut-off Frequency:
As both T and π sections have same cut-off frequency, it is sufficient to calculate the cut-off frequency for the T section only.

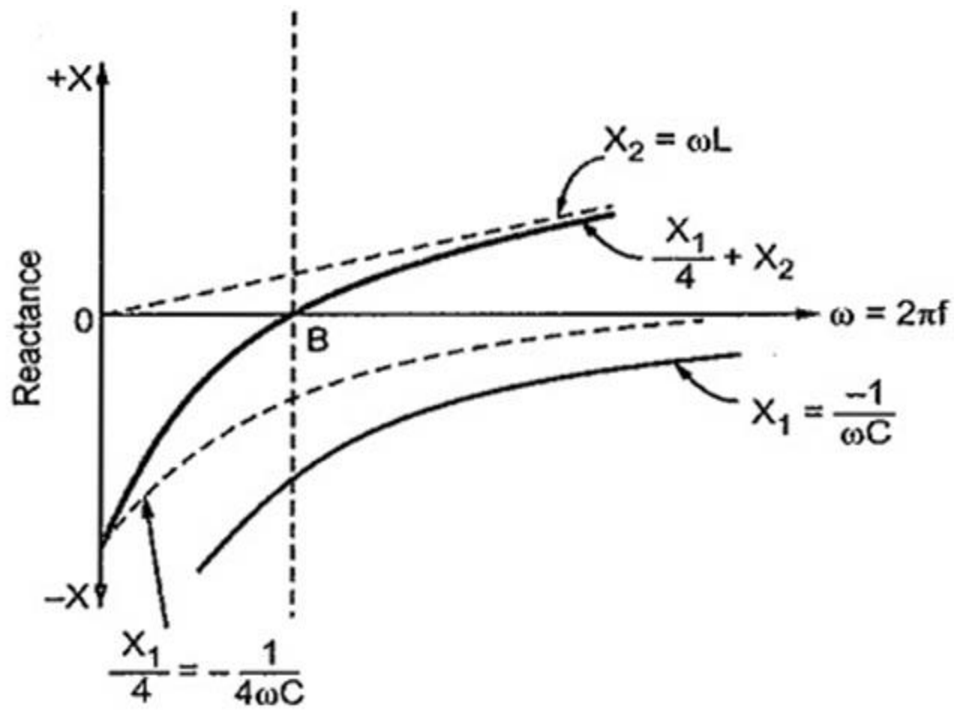


Fig 7 Reactance frequency Sketch

The reactance curves are as shown in the Fig. 7.
From above characteristics it is clear that all the reactance curves have positive slope as all curves slope upward to the right side with increasing ω.
Here the curves are on the same side of the horizontal axis up to the point B, giving a stop band. For frequencies above point B, the curves are on opposite sides of the axis, giving pass band. Thus, point B gives cut-off frequency, represented as $\omega = \omega_c$.
At point B, $\omega = \omega_c$, the curve for $(X_1/4 + X_2)$ crosses the frequency axis, hence we can write,

$$\omega_c L - \frac{1}{4\omega_c C} = 0$$

$$\omega_c L = \frac{1}{4\omega_c C}$$

$$\omega_c^2 = \frac{1}{4LC} \qquad \dots (2)$$

$$\omega_c = \frac{1}{2\sqrt{LC}} \quad \text{or}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}} \qquad \dots (3)$$

gebraic approach to calculate cut-off frequency is as follows.

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{-1}{4\omega^2 C^2} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}} \times \sqrt{1 - \frac{1}{4\omega^2 LC}}$$

$$Z_{0T} = R_0 \sqrt{1 - \frac{1}{4\omega^2 LC}} \qquad \text{... (4)}$$

From above expression it is clear that, Z_{0T} is real if $1/4\omega^2 LC < 1$ and imaginary if $1/4\omega^2 LC >$
Hence condition $1 - 1/4\omega^2 LC = 0$ gives expression,

$$\omega^2 = \frac{1}{4LC} \text{ or } \omega = \frac{1}{2\sqrt{LC}}$$

Thus, above prototype section passes all frequencies above $\omega = 1/2\sqrt{LC}$ while attenuates a
value. Therefore cut-off frequency of high pass filter is given by

$$\omega_c = \frac{1}{2\sqrt{LC}} \text{ or } f_c = \frac{1}{4\pi\sqrt{LC}}$$

quency comes out to be same as frequency calculated by reactance sketch method.

Variation of Z_{0T} and $Z_{0\pi}$ with Frequency:

Consider expression for Z_{0T} as

$$Z_{0T} = R_0 \sqrt{1 - \frac{1}{4\omega^2 LC}}$$

From equation (2) we can write,

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega_c^2}{\omega^2}} \quad \text{or}$$

$$Z_{0T} = R_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad \dots (5)$$

Similarly we can write,

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{R_0^2}{R_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Hence,

$$Z_{0\pi} = \frac{R_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \qquad \dots (6)$$

From equation (5), it is clear that as frequency f increases from f_c to ∞ in pass band, Z_{0T} also increases from 0 to R_0.

For π section, from equation (6), it is clear that as frequency increases from f_c to ∞, Z_0 decreases from ∞ to R_0 in pass band. The variation of Z_{0T} and $Z_{0\pi}$ with frequency is as shown in Fig. 8

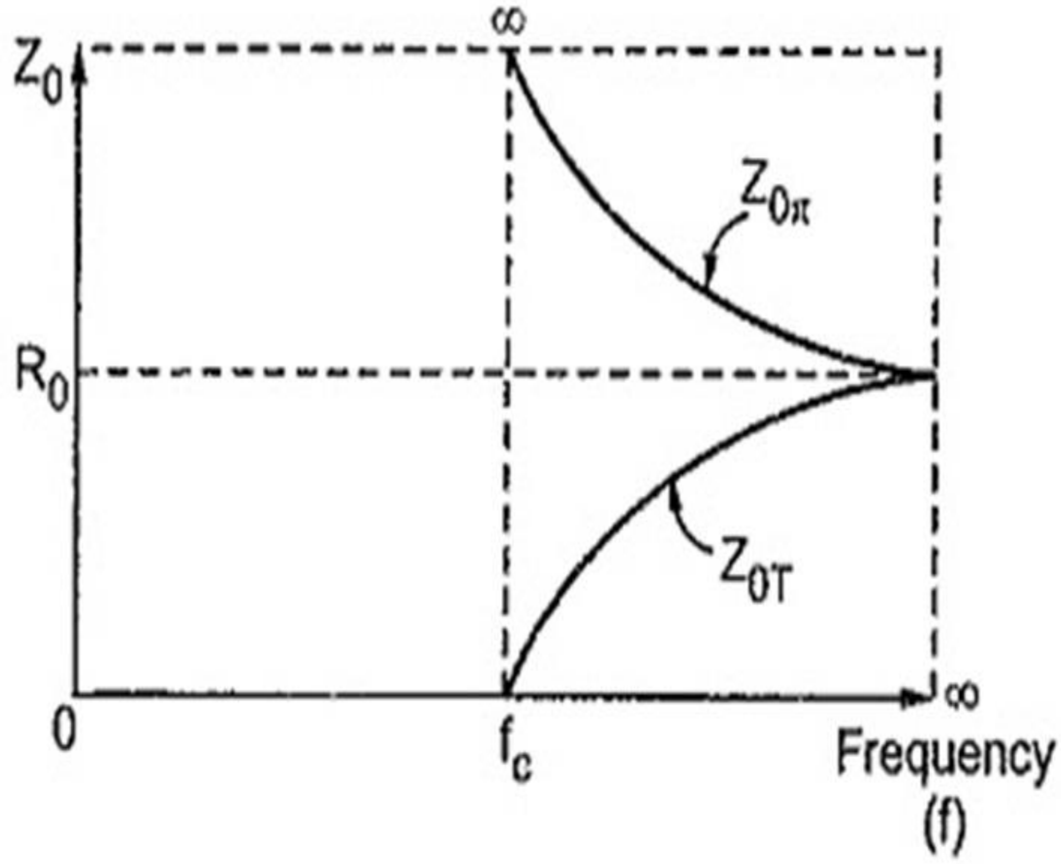


Fig 8 Variation of Characteristics impedance with frequency

Variation of Attenuation Constant (α) with Frequency:

In pass band, attenuation is zero ($\alpha = 0$). In stop band attenuation is given by

$$\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right) \qquad \dots (7)$$

In stop band, as frequency f increases from 0 to f_c, attenuation decreases from ∞ to 0. The constant α with frequency is as shown in the Fig. 9

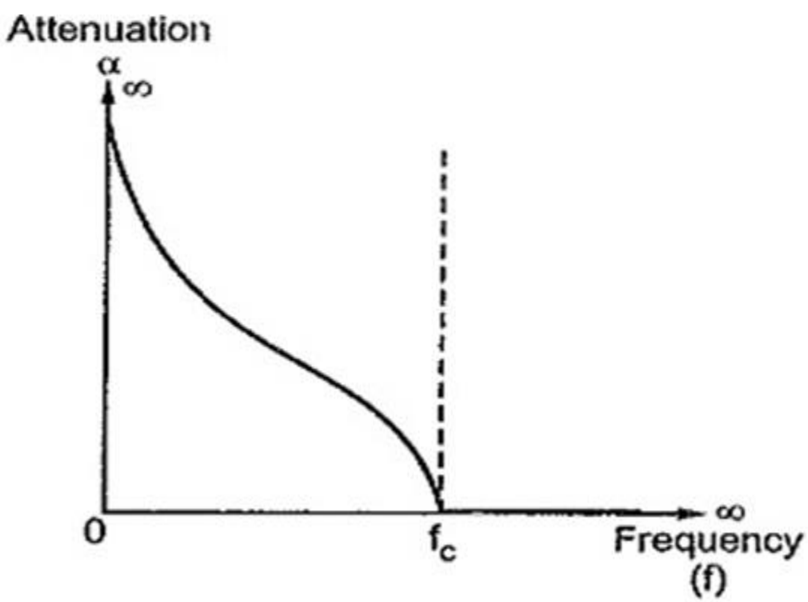


Fig 9 Variation of attenuation constant with frequency

Variation of Phase Constant β with Frequency:
In stop band, phase constant β is always π radian. In pass band where α = 0, the phase angle β is given by

$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right) \quad \ldots (8)$$

From the above equation it is clear that in pass band when frequency f increases from f_c to ∞, β decre
constant β with frequency is as shown in the Fig. 10.

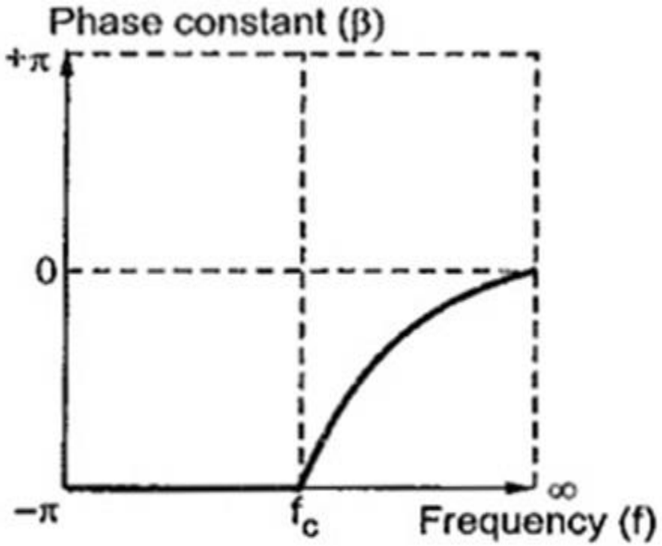


Fig 10 Variations of phase constant with frequency

Design Equations of Prototype High Pass Filter:

The design impedance R_0 and cut-off frequency f_c for high pass filter section can be given in terms o

$$R_0 = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

Dividing equation for R_0 by f_c, we get,

$$L = \frac{R_0}{(4\pi f_c)} \qquad \text{... (9)}$$

Multiplying equation for R_0 and f_c, we get,

$$C = \frac{1}{(4\pi f_c)R_0} \qquad \text{... (10)}$$

n (9) and (10) are called design equations of prototype high pass filter sections.

Band Pass Filter:

pass filter pass a certain range of frequencies
ed as **pass band**) while attenuate all other frequencies.
band pass filters can be obtained by connecting
pass filter sections in cascade with high pass filter sectio
own in Fig. 11.

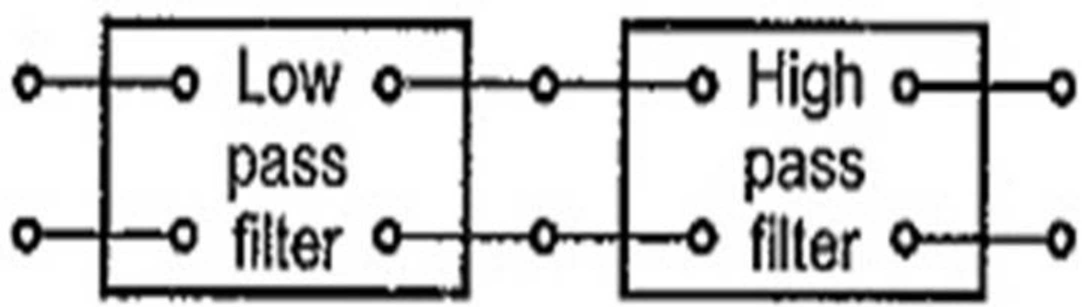


Fig 11

In above type of connection, the cut-off frequency of low pass filter section must
that of high pass filter section. Although cascade connection of low pass filter and
functions properly as band pass filter, it is more economical to combine both sect
section. An alternative form of band pass filter can be obtained either as a T or π
contains a series resonant circuit while the shunt arm contains a parallel resonan
shown in the Fig. 12 (a) and (b).

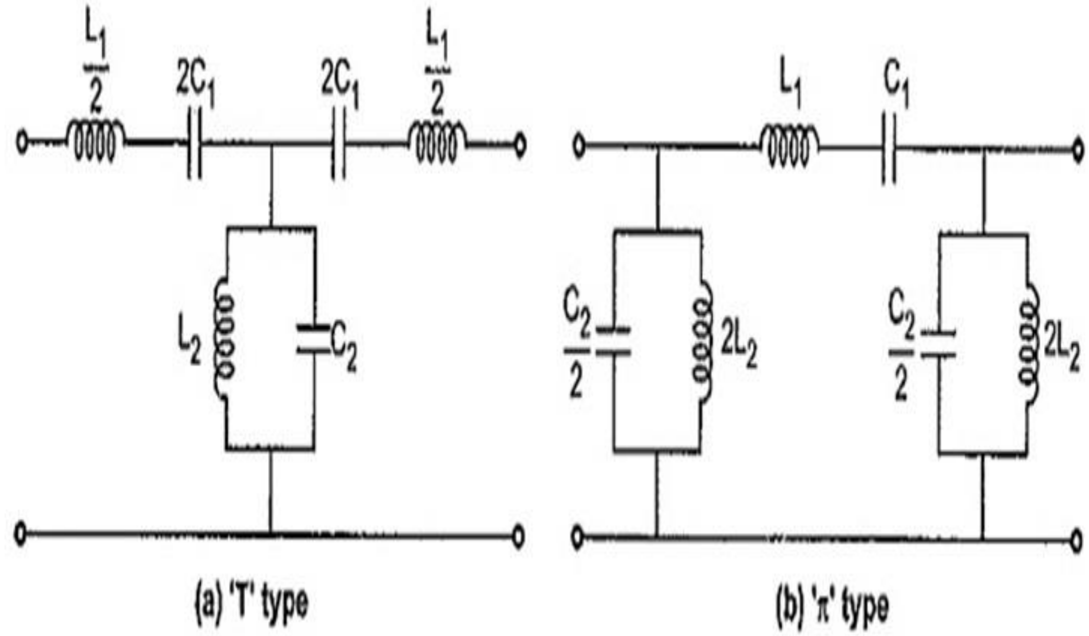


Fig 12 Prototype T and
π *band pass filter*

The band pass filter characteristics can be obtained by using conventional band pass filter (either T or π type) as shown in the Fig. 9.16, if the series resonant frequency of the series arm is selected same as anti resonant frequency of the shunt arm. Consider T type band pass filter section as shown in the Fig. 9.16 (a). Let the frequency of series and shunt arm be ω_0 rad/sec.

Then, for series arm, frequency of resonance is given by,

$$\omega_0 = \frac{1}{\sqrt{\left(\frac{L_1}{2}\right)(2C_1)}} = \frac{1}{\sqrt{L_1 C_1}}$$

$$\omega_0^2 L_1 C_1 = 1 \qquad \text{... (1)}$$

Similarly for shunt arm, frequency of anti resonance is given by,

$$\omega_0 = \frac{1}{\sqrt{(L_2)(C_2)}}$$

$$\omega_0^2 L_2 C_2 = 1 \qquad \text{... (2)}$$

From equations (1) and (2), for same resonant frequencies of series and shunt arms we can write,

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$

$$L_1 C_1 = L_2 C_2 \qquad \text{... (3)}$$

Design Impedance (R_0):

Total series arm impedance Z_1 is given by

$$Z_1 = j\omega L_1 + \left(\frac{-j}{\omega C_1}\right) = j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right] \qquad \text{... (4)}$$

Similarly, total shunt arm impedance Z_2 is given by

$$Z_2 = (j\omega L_2) \,||\, \left(\frac{-j}{\omega C_2}\right) = \frac{(j\omega L_2)\left(\frac{-j}{\omega C_2}\right)}{j\omega L_2 - \frac{j}{\omega C_2}}$$

$$Z_2 = \frac{\frac{L_2}{C_2}}{\frac{j(\omega^2 L_2 C_2 - 1)}{\omega C_2}}$$

$$Z_2 = \frac{-j\omega L_2}{(\omega^2 L_2 C_2 - 1)} \qquad \text{... (5)}$$

Hence, $Z_1 Z_2 = L_2/C_1 = L_1/C_2$ which is real and constant. Hence above sections are constant k sections.

o we can write,

$$R_0^2 = Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2}$$

$$R_0 = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}} \qquad \dots (6)$$

Reactance Curves and Expressions for Cut-off Frequencies:

To verify the band pass characteristics, let $Z_1 = j X_1$ and $Z_2 = j X_2$. Similar to the reactance curves drawn for low pass filter section and high pass filter section, sketching reactances X_1 and $(X_1/4 + X_2)$ against frequency f as shown in the Fig. 13.

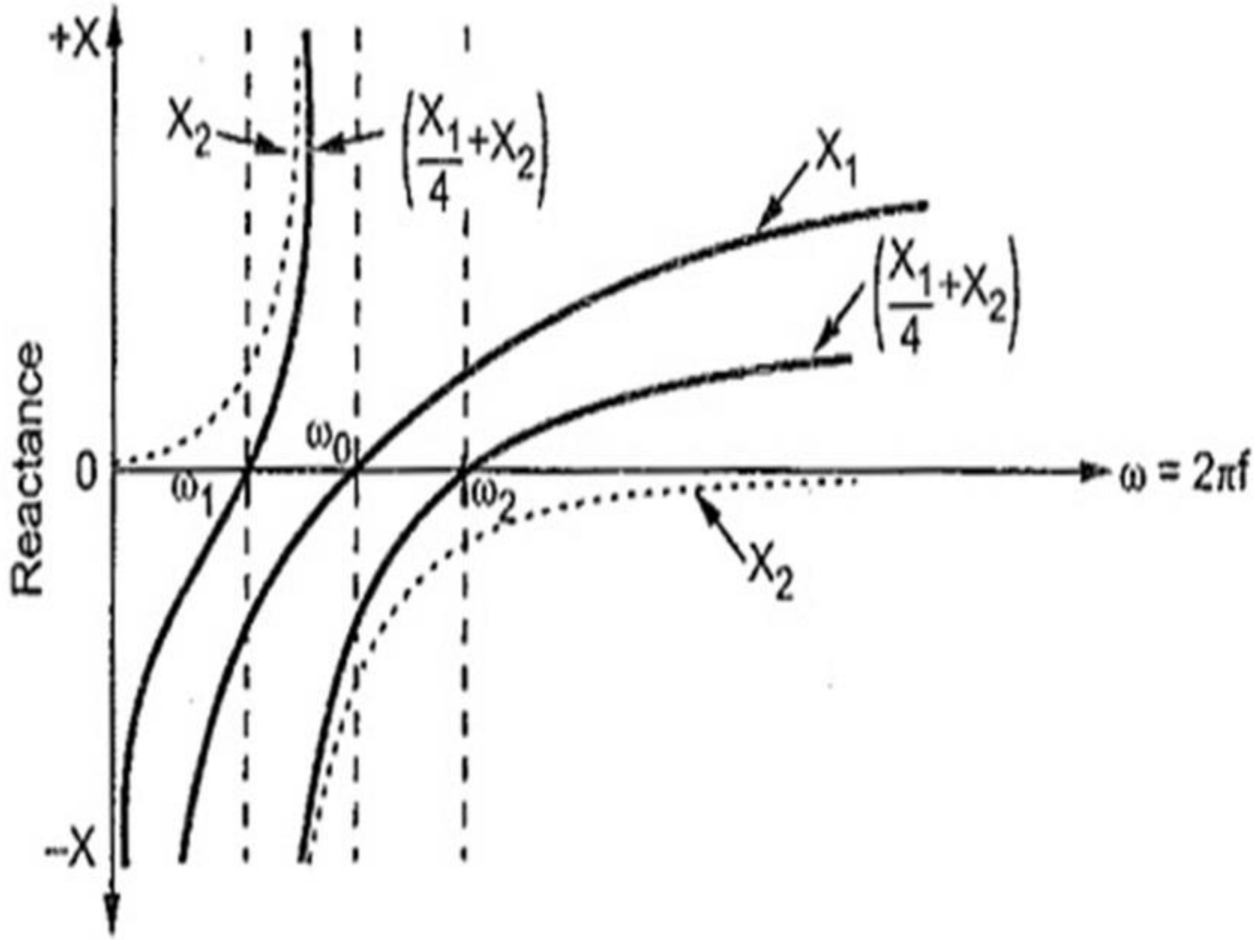


Fig 13 Reactance frequency sketch

$(X_1/4 + X_2)$ are on the
same sides the axis below f_1 and above f_2. At the same time, the reactance curves between f_1 and f_2 are
on opposite sides of frequency axis. Thus frequencies between f_1 and f_2 constitute a pass band ; while
the frequencies below f_1 and above f_2 give stop band. Hence the section considered shows band pass filter
characteristics where f_1 and f_2 are lower and upper cut-off frequencies of the filter.

In band pass filter, condition for cut-off frequency is,

$$\frac{Z_1}{4} + Z_2 = 0$$

$$Z_1 = -4\,Z_2$$

$$Z_1^2 = -4\,Z_1 Z_2$$

But from the condition of constant-k filter section, $Z_1 Z_2 = R^2_0$

$$Z_1^2 = -4\,R_0^2$$

$$Z_1 = \pm j\,(2\,R_0) \qquad \ldots (7)$$

From above equation (7) it is clear that the value of the series arm impedance Z_1 can be o different cut-off frequencies namely f_1 and f_2. So at $f = f_1$, $Z_1 = -j(2 R_0)$ and at $f = f_2$, $Z_1 =$ Thus impedance Z_1 at f_1, i.e. lower cut-off frequency, is negative of the impedance Z_1 at f_2 i.e. upper cut-off frequency. Hence we can write,

$$\omega_2 L_1 - \frac{1}{\omega_2 C_1} = -\left(\omega_1 L_1 - \frac{1}{\omega_1 C_1}\right)$$

$$\frac{\omega_2^2 L_1 C_1 - 1}{\omega_2 C_1} = -\left(\frac{\omega_1^2 L_1 C_1 - 1}{\omega_1 C_1}\right)$$

$$\omega_2^2 L_1 C_1 - 1 = \frac{\omega_2}{\omega_1}\left(1 - \omega_1^2 L_1 C_1\right) \qquad \dots (8)$$

ut from equation (1) we can write,

$$\omega_0^2 = \frac{1}{L_1 C_1} \quad \text{or} \quad L_1 C_1 = \frac{1}{\omega_0^2}$$

Substituting value of (L_1 C_1) in above equation (8), we can write,

$$\frac{\omega_2^2}{\omega_0^2} - 1 = \frac{\omega_2}{\omega_1}\left(1 - \frac{\omega_1^2}{\omega_0^2}\right)$$

$$\left(1 - \frac{\omega_1^2}{\omega_0^2}\right) = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2}{\omega_0^2} - 1\right)$$

Simplifying above equation,

$$\frac{\omega_0^2 - \omega_1^2}{\omega_0^2} = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2 - \omega_0^2}{\omega_0^2}\right)$$

$$\omega_2\,\omega_0^2 - \omega_2\,\omega_1^2 = \omega_1\,\omega_2^2 - \omega_1\,\omega_0^2$$

$$\omega_2\,\omega_0^2 + \omega_1\,\omega_0^2 = \omega_1\,\omega_2^2 + \omega_2\,\omega_1^2$$

$$\omega_0^2\,(\omega_2 + \omega_1) = \omega_1\,\omega_2\,(\omega_2 + \omega_1)$$

$$\omega_0^2 = \omega_1\,\omega_2$$

$$f_0^2 = f_1 \cdot f_2$$

$$f_0 = \sqrt{f_1 \cdot f_2} \qquad \text{... (9)}$$

Hence, above equation (9) indicates that frequency of resonance of the individual arms is the geometric mean of two cut-off frequencies:

Variation of Z_{0T} and $Z_{0\pi}$, Attenuation Constant (α) and Phase Constant (β) with Frequency:

The variations of Z_{0T} and $Z_{0\pi}$, attenuation constant (α) and phase shift (β) with frequency are as shown in the Fig. 14. (a), (b) and (c). Consider that the design impedance of band pass filter is R_0 and cut-off frequencies are f_1 and f_2.

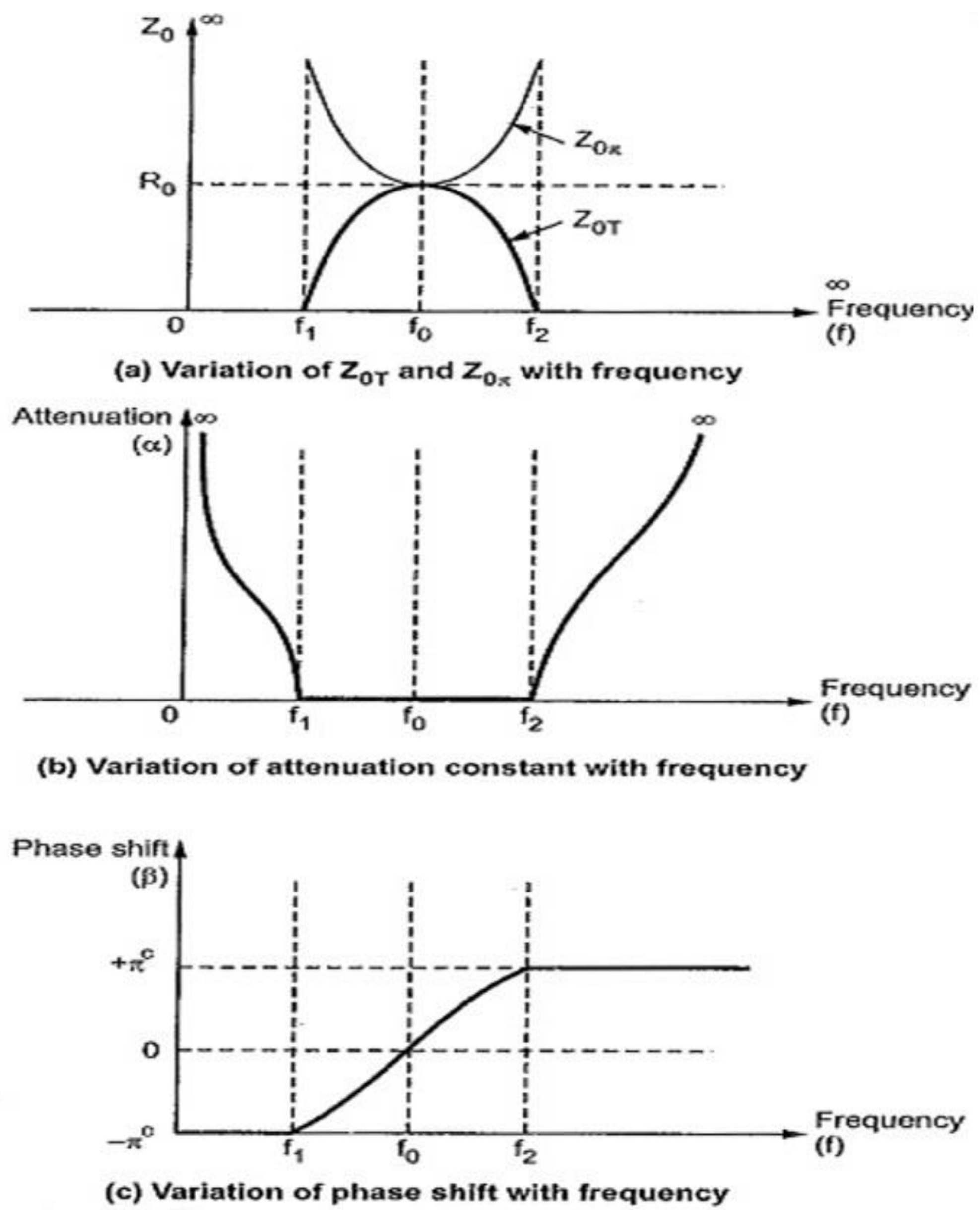


Fig 14 Variation of phase shift with f

Design Equations:

Consider that the filter is terminated in design impedance R_0 and the cut-off frequencies are f_1 and f_2.

Then from equation (7), at the lower cut-off frequency f_1, we can write,

$$j\omega_1 L_1 - \frac{j}{\omega_1 C_1} = -j(2R_0)$$

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = 2R_0$$

$$1 - \omega_1^2 L_1 C_1 = 2R_0(\omega_1 C_1)$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = (2\omega_1 C_1) R_0 \; \ldots\ldots \; \because \; \omega_0^2 = \frac{1}{L_1 C_1}$$

$$1 - \frac{(2\pi f_1)^2}{(2\pi f_0)^2} = 2(2\pi f_1) C_1 R_0$$

$$1 - \frac{f_1^2}{f_0^2} = 4\pi R_0 f_1 C_1$$

$$f_0^2 = f_1 f_2$$

$$1 - \frac{f_1^2}{f_1 f_2} = 4\pi R_0 f_1 C_1$$

$$\frac{f_2 - f_1}{f_2} = 4\pi R_0 f_1 C_1$$

$$C_1 = \frac{(f_2 - f_1)}{4\pi R_0 (f_1 f_2)} \qquad \ldots (10)$$

But for band pass filter constant k section

$$f_0 = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

$$f_0^2 = \frac{1}{4\pi^2 L_1 C_1}$$

$$L_1 = \frac{1}{4\pi^2 C_1 f_0^2}$$

Substituting the value of C_1 from equation (10),

$$L_1 = \frac{1}{4\pi^2 f_0^2 \left[\dfrac{(f_2 - f_1)}{4\pi R_0 f_2 f_1}\right]}$$

As $f^2_0 = f_1 f_2$, we get

$$L_1 = \frac{R_0}{\pi(f_2 - f_1)} \qquad \text{... (11)}$$

rom equation (6), we can write,

$$R_0^2 = \frac{L_1}{C_2}$$

$$C_2 = \frac{L_1}{R_0^2}$$

Substituting value of L_1 from equation (11),

$$C_2 = \frac{\pi(f_2 - f_1)}{R_0^2}$$

$$C_2 = \frac{1}{\pi R_0 (f_2 - f_1)} \qquad \text{... (12)}$$

From equation.(6), we can write,

$$R_0^2 = \frac{L_2}{C_1}$$

$$L_2 = C_1 \cdot R_0^2$$

Substituting value of C_1 from equation (10),

$$L_2 = \frac{(f_2 - f_1)}{4\pi R_0 (f_1 f_2)} R_0^2$$

$$L_2 = \frac{R_0 (f_2 - f_1)}{4\pi f_1 f_2} \qquad \text{... (13)}$$

Equations (10) to (13) are called design equations of prototype band pass filter sections.

Band Stop Filter:

Band Stop Filter stop a range of frequencies between two cut-off frequencies f_1 and f_2 while pass all the frequencies below f_1 and above f_2. Thus range of frequencies between f_1 and f_2 constitutes a stop band in which attenuation to the frequencies is infinite ideally. The frequencies below f_1 and above f_2 constitute two separate pass bands in which attenuation to the frequencies is zero ideally.

The Band Stop Filter can be obtained by connecting low pass filter and high pass filter sections in parallel where cut-off frequency of the low pass filter section is less than that of the high pass filter section. But the economical form of the band elimination filter can be obtained by combining the low pass and high pass

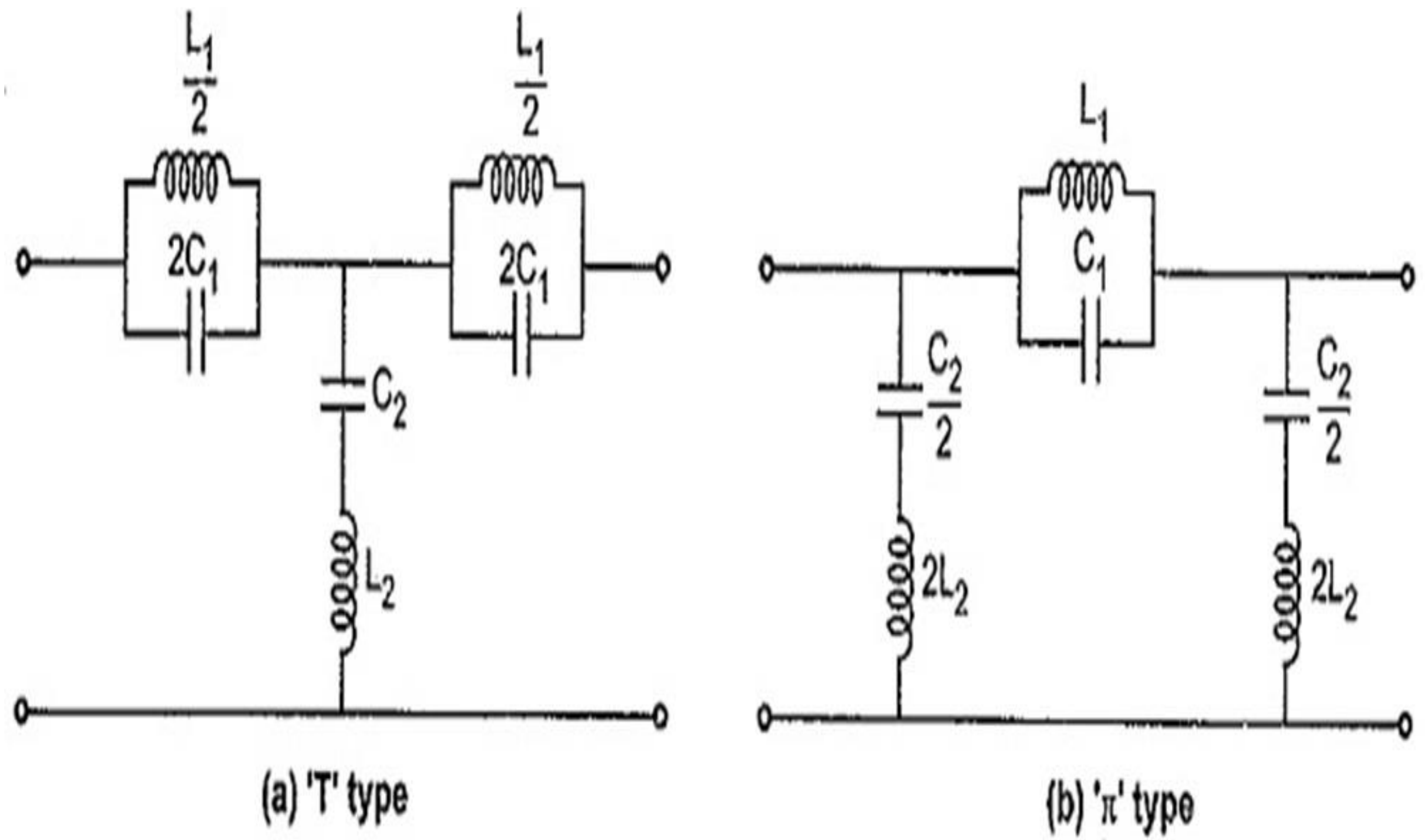


Fig 15 Prototype Band pass filter

The band elimination characteristics can be obtained by using conventional Band Stop Filter (either T o
if the series resonant frequency of the shunt arm is selected same as the parallel resonant frequency o
Consider 'T' type band elimination filter section as shown in the Fig. 15(a).Let the frequency of the serie
Then for series arm, frequency of anti resonance is given by,

$$\omega_0 = \frac{1}{\sqrt{\left(\frac{L_1}{2}\right)(2C_1)}} = \frac{1}{\sqrt{L_1 C_1}}$$

$$\omega_0^2 L_1 C_1 = 1 \quad \ldots (1)$$

Similarly, for shunt arm, frequency of resonance is given by,

$$\omega_0 = \frac{1}{\sqrt{(L_2)(C_2)}}$$

$$\omega_0^2 L_2 C_2 = 1 \quad \ldots (2)$$

rom equations (1) and (2), for same resonant frequencies of series and shunt arm resonan

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$

$$L_1 C_1 = L_2 C_2 \quad \ldots (3)$$

Design Impedance (R_0):

Total series arm impedance is given by,

$$Z_1 = \frac{(j\omega L_1)\left(-\frac{j}{\omega C_1}\right)}{j\omega L_1 - \frac{j}{\omega C_1}} = \frac{\omega L_1}{j\left(\omega^2 L_1 C_1 - 1\right)}$$

$$Z_1 = \frac{-j\omega L_1}{\left(\omega^2 L_1 C_1 - 1\right)}$$

$$Z_1 = \frac{\omega L_1}{\left(1 - \omega^2 L_1 C_1\right)} \qquad \text{... (4)}$$

Similarly, total shunt arm impedance Z_2 is given by,

$$Z_2 = j\omega L_2 - \frac{j}{\omega C_2} = j\left(\frac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right) \qquad \text{... (5)}$$

Hence $Z_1Z_2 = L_2/C_1 = L_2/C_2$ which is real and constant. Hence above sections are constant k

$$R_0^2 = Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$$

$$R_0 = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}} \qquad \text{... (6)}$$

Reactance Curves and Expressions for Cut-off Frequencies:

To verify the band elimination characteristics, let Z_1 =j X_1 and Z_2 =j X_2. The reactance curves of X_1 and $X_1/4 + X_2$ against frequency are as shown in the Fig. 16

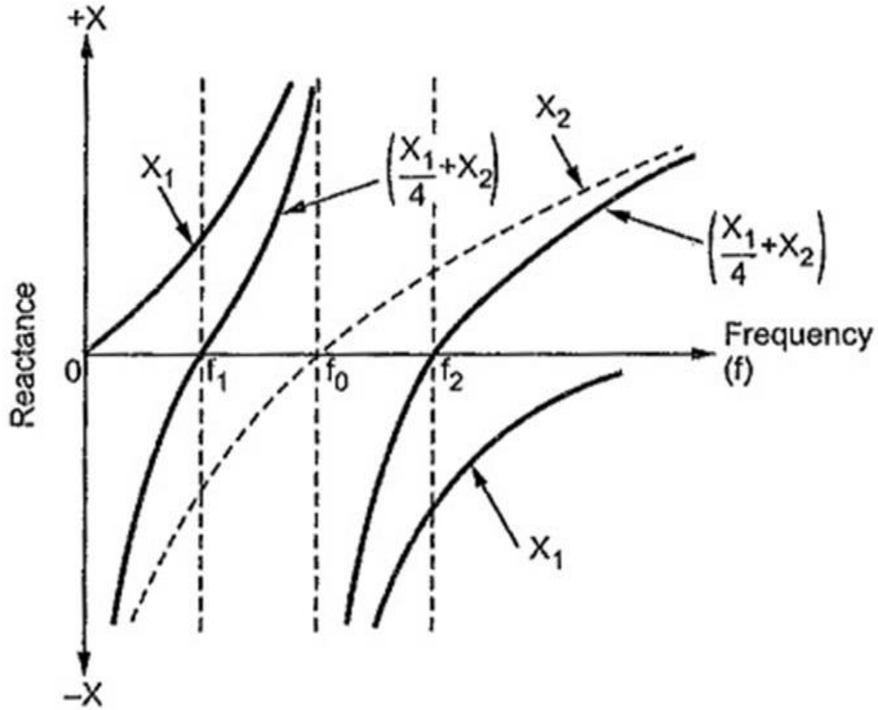


Fig 16 Reactance frequency sketch

From the above characteristics it is clear that the reactance curves for X_1 and $(X_1/4 + X_2)$ ar
frequency axis between f_1 and f_2 which indicates stop band.

These curves are on opposite sides of the axis below f_1 and above f_2 which indicates two pa
Hence for the given section, the characteristics are of band elimination filter where f_1 and f
In Band Stop Filter, the condition for cut-off frequencies is given by

$$\frac{Z_1}{4} + Z_2 = 0$$

$$\frac{Z_1}{4} = -Z_2$$

$$Z_1^2 = -4\, Z_1\, Z_2$$

But from the condition of constant K filter section, $Z_1 Z_2 = R^2_0$

$$Z_1^2 = -4 R_0^2$$

$$Z_1 = \pm j\,(2\, R_0) \qquad \dots (7)$$

From above equation it is clear that the value of the series arm impedance Z_1 can be obtained at two different cut-off frequencies namely f_1 and f_2. So at $f = f_1$, $Z_1 = +\, j(2\, R_0)$ and at $f = f_2$, $Z_1 = -j(2\, R_0)$. Thus impedance Z_1 at f_1, i.e. at lower
cut-off frequency, is negative of the impedance Z_1 at f_2 i.e. upper cut-off frequency. Hence we can write,

$$\frac{\omega_1 L_1}{1-\omega_1^2 L_1 C_1} = -\frac{\omega_2 L_1}{1-\omega_2^2 L_1 C_1}$$

$$1-\omega_2^2 L_1 C_1 = \frac{-\omega_2}{\omega_1}\left(1-\omega_1^2 L_1 C_1\right)$$

$$1-\omega_2^2 L_1 C_1 = \frac{\omega_2}{\omega_1}\left(\omega_1^2 L_1 C_1 - 1\right) \quad \text{... (8)}$$

But from equation (1), frequency of resonance is given by

$$\omega_0^2 = \frac{1}{L_1 C_1} \quad \text{or} \quad L_1 C_1 = \frac{1}{\omega_0^2}$$

Substituting value of (L_1 C_1) in above equation, we can write,

$$1-\frac{\omega_2^2}{\omega_0^2} = \frac{\omega_2}{\omega_1}\left(\frac{\omega_1^2}{\omega_0^2}-1\right)$$

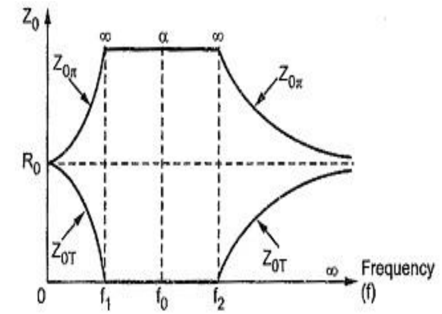
Simplifying above equation,

$$
\begin{aligned}
\frac{\omega_0^2 - \omega_2^2}{\omega_0^2} &= \frac{\omega_2}{\omega_1}\left(\frac{\omega_1^2 - \omega_0^2}{\omega_0^2}\right) \\
\omega_1\,\omega_0^2 - \omega_1\,\omega_2^2 &= \omega_2\,\omega_1^2 - \omega_2\,\omega_0^2 \\
\omega_1\,\omega_0^2 + \omega_2\,\omega_0^2 &= \omega_2\,\omega_1^2 + \omega_1\,\omega_2^2 \\
\omega_0^2\left(\omega_1 + \omega_2\right) &= \omega_1\,\omega_2\left(\omega_2 + \omega_1\right) \\
\omega_0^2 &= \omega_1\,\omega_2 \\
f_0^2 &= f_1\,f_2 \\
f_0 &= \sqrt{f_1\;f_2} \qquad \ldots (9)
\end{aligned}
$$

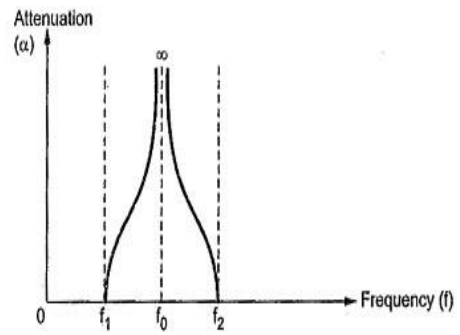
Hence, above equation (9) indicates that in band elimination filter, the frequency of resonance of the individual arms is the geometric mean of two cut-off frequencies.

Variation of Z_{0T} and $Z_{0\pi}$, Attenuation Constant (α), Phase Constant (β) with Frequency:

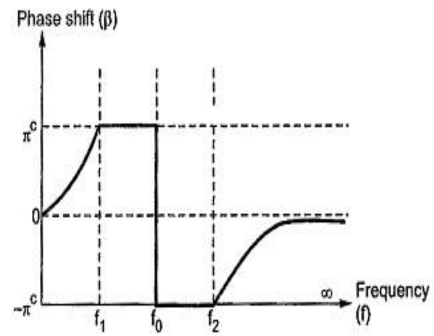
The variations of Z_{0T} and $Z_{0\pi}$, attenuation constant (α) and phase shift (β) are as shown in the Fig. 9.22 (a), (b) and (c) respectively. Consider that f_1 and f_2 are two cut-off frequencies and R_0 is the design impedance of the Band Stop Filter.



(a) Variation of Z_{0T} and $Z_{0\pi}$ with frequency



(b) Variation of attenuation constant (α) with frequency



(c) Variation of phaseshift (β) with frequency

Fig 17

Design Equations:

Consider that a band elimination filter with two cut-off frequencies f_1 and f_2 is terminated in design impedance R_0. Then, from equation (7), at lower cut-off frequency f_1, we can write,

$$j\left(\frac{\omega_1 L_1}{1-\omega_1^2 L_1 C_1}\right) = +j(2R_0)$$

$$\omega_1 L_1 = 2R_0\left(1-\omega_1^2 L_1 C_1\right)$$

$$\omega_1 L_1 = 2R_0\left(1-\frac{\omega_1^2}{\omega_0^2}\right) \text{.........} \because \quad L_1 C_1 = \frac{1}{\omega_0^2}$$

$$\omega_1 L_1 = 2R_0\left(1-\frac{\omega_1^2}{\omega_1 \omega_2}\right) \text{..........} \omega_0^2 = \omega_1 \omega_2$$

$$L_1 = 2R_0\left(\frac{\omega_2-\omega_1}{\omega_1 \omega_2}\right)$$

$$L_1 = \frac{2R_0(2\pi)(f_2-f_1)}{4\pi^2(f_1 f_2)}$$

$$L_1 = \frac{R_0(f_2-f_1)}{\pi f_1 f_2} \qquad \text{... (10)}$$

For band elimination filter constant K section, frequency of resonance in series ar

$$f_0 = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

$$f_0^2 = \frac{1}{4\pi^2 L_1 C_1}$$

$$C_1 = \frac{1}{4\pi^2 f_0^2 L_1} = \frac{1}{4\pi^2 (f_1 f_2) L_1} \quad \cdots\cdots\cdots \because \quad f_0^2 = f_1 f_2$$

Substituting value of L_1 in above equation,

$$C_1 = \frac{1}{4\pi^2 (f_1 f_2)\left[\dfrac{R_0 (f_2 - f_1)}{\pi(f_1 f_2)}\right]}$$

$$C_1 = \frac{1}{4\pi R_0 (f_2 - f_1)} \qquad \ldots (11)$$

From equation (6) we can write,

$$R_0^2 = \frac{L_1}{C_2}$$

$$C_2 = \frac{L_1}{R_0^2}$$

Substituting value of L_1 from equation (10),

$$C_2 = \frac{1}{R_0^2}\left[\frac{R_0(f_2 - f_1)}{\pi(f_1\, f_2)}\right]$$

$$C_2 = \frac{(f_2 - f_1)}{\pi R_0(f_1\, f_2)} \qquad \text{... (12)}$$

Similarly from equation (6) we can write,

$$R_0^2 = \frac{L_2}{C_1}$$

$$L_2 = R_0^2\; C_1$$

Substituting value of C_1 from equation (11),

$$L_2 = R_0^2\left[\frac{}{4\pi R_0(f_2 - f_1)}\right]$$

$$L_2 = \frac{R_0}{4\pi(f_2 - f_1)} \qquad \text{... (13)}$$

Equations (10) to (13) are called design equations of prototype band elimination filter sections.

m Derived Filters:

m Derived Filters – The first disadvantage of prototype filter sections can be overcome by connecting two or more prototype sections of same type (either all T type or all π type) in cascade. In such a cascade connection, attenuation to the frequencies in pass band remains zero ideally, but attenuation to the frequencies in entire attenuation band considerably increases. e.g. If two sections of same type are cascaded, the attenuation in the attenuation band gets doubled giving much sharper cut-off characteristics than that obtained by using only a single section.

But due to the resistance in the components used in cascade connection, the attenuation in pass band slightly increases, instead of being zero. Thus the curve

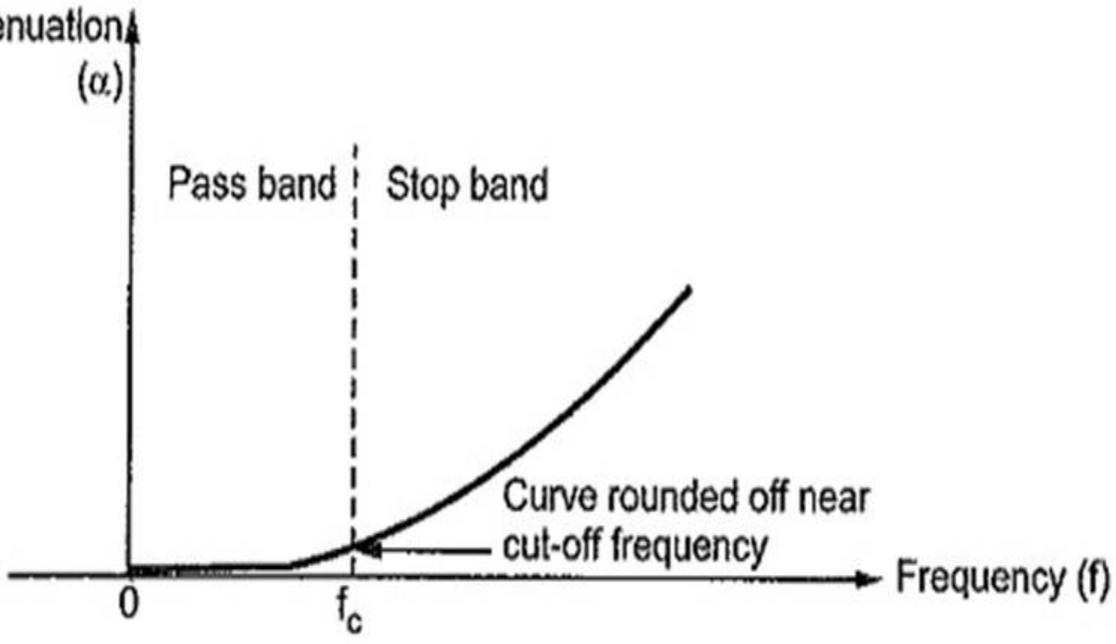


Fig 18 Attenuation constant with frequency

To fulfill all the requirements discussed above, it is necessary to design a new section having same cut-off frequency as that of the prototype section but different attenuation characteristics in the attenuation band. Also to maintain same cut-off frequency, both the sections must have same characteristic impedance Z_0. It is possible to derive a new section from a prototype constant K section. Thus, a new section derived is called **m-derived section**.

Derivation of m-derived Sections:

Consider any general T section and a new section derived from it as shown in the Fig. 19 (a) and (b).

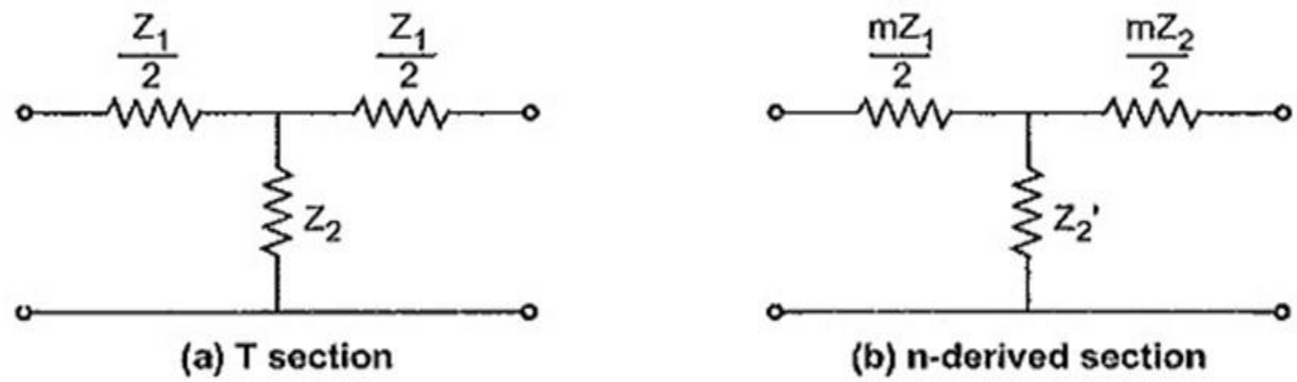


Fig 19

In the m-derived section, the series arm is of same type as that in prototype section but having different value i.e. m $Z_1/2$ where m is a constant. Now Z_2 of prototype section will change to Z'_2 in m-derived section such that the value of Z_0 for both the sections is same.

For a prototype section, the characteristic impedance is given by,

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \qquad \text{... (1)}$$

Similarly for a m-derived section, the characteristic impedance is given by,

$$Z_{0T} = \sqrt{\frac{(m Z_1)^2}{4} + (m Z_1) Z_2'} \qquad \text{... (2)}$$

To maintain same Z_0, equating equations (1) and (2) by squaring,

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \frac{m^2 Z_1^2}{4} + m Z_1 Z_2'$$

$$m Z_1 Z_2' = \frac{Z_1^2}{4} - \frac{m^2 Z_1^2}{4} + Z_1 Z_2$$

$$m Z_1 Z_2' = \frac{(1-m^2)}{4} Z_1^2 + Z_1 Z_2$$

$$Z_2' = \left(\frac{1-m^2}{4m}\right) Z_1 + \frac{Z_2}{m} \qquad \text{... (3)}$$

From equation (3) it is dear that, the shunt arm of m-derived section is a series connection of two impedances (Z_2/m) and ($1-m^2/4m$) Z

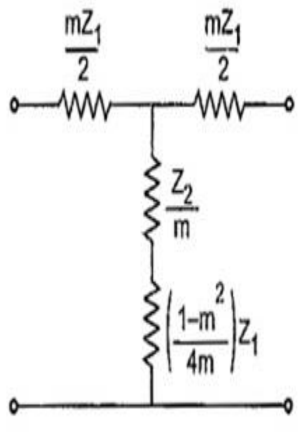


Fig 20 m-Derived T section

The same technique can be used to obtain m-derived π section. Consider any general π section with shunt arm of same type but having different value i.e. Z_2/m as shown in the Fig. 21.

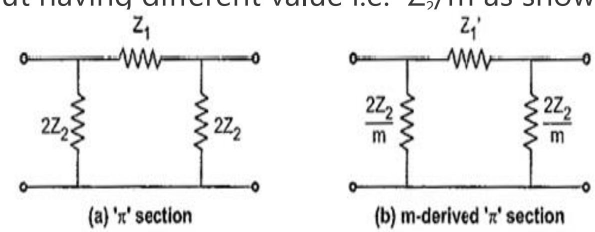


Fig 21

For a prototype π section, the characteristic impedance $Z_{0\pi}$ is given by,

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4 Z_2}}} \quad \text{... (4)}$$

Similarly for a m-derived π section, the characteristic impedance is given by,

$$Z_{0\pi} = \sqrt{\frac{Z_1'\left(\frac{Z_2}{m}\right)}{1+\frac{Z_1'}{4\left(\frac{Z_2}{m}\right)}}}$$

$$Z_{0\pi} = \sqrt{\frac{\frac{Z_1' Z_2}{m}}{1+\frac{m Z_1'}{4 Z_2}}} \qquad \ldots (5)$$

Squaring equations (4) and (5) and equating,

$$\frac{Z_1 Z_2}{1+\frac{Z_1}{4 Z_2}} = \frac{\frac{Z_1 Z_2}{m}}{1+\frac{m Z_1'}{4 Z_2}}$$

$$\therefore\ Z_1 Z_2 + Z_1'\left(\frac{m Z_1}{4}\right) = Z_1'\left(\frac{Z_2}{m}+\frac{Z_1}{4\,m}\right)$$

$$\therefore\ Z_1'\left[\frac{Z_2}{m}+\frac{Z_1}{4\,m}-\frac{m Z_1}{4}\right] = Z_1 Z_2$$

$$\therefore\ Z_1'\left[\frac{Z_2}{m}+\frac{Z_1}{4\,m}\left(1-m^2\right)\right] = Z_1 Z_2$$

$$\therefore \qquad Z_1' = \frac{Z_1 Z_2}{\frac{Z_2}{m}+\left(\frac{1-m^2}{4\,m}\right) Z_1}$$

$$Z_1' = \frac{(m Z_1)(4 Z_2)}{4 Z_2+\left(1-m^2\right) Z_1}$$

Multiplying numerator and denominator by the factor m/1-m² we get

$$Z_1' = \frac{(m Z_1)\left(\frac{4m}{1-m^2}\right) Z_2}{\left(\frac{4m}{1-m^2}\right) Z_2 + m Z_1} \quad \text{... (6)}$$

From equation (6) it is clear that, the series arm Z'_1 of m-derived section is a parallel combination of t impedances ($m Z_1$) and ($4m/1-m^2$) Z_2 with condition $0 < m < 1$. The m-derived π section is as shown i

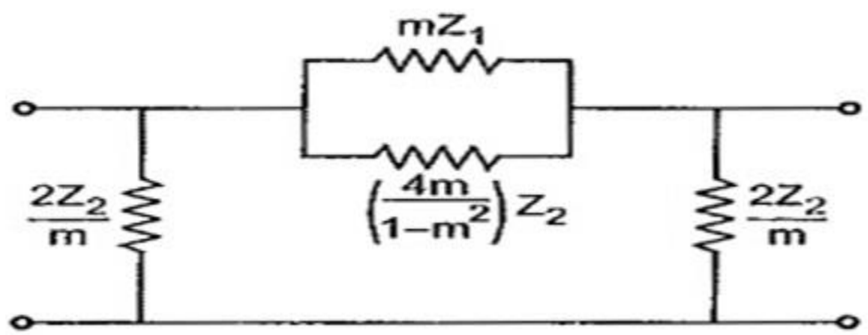


Fig 22 m-Derived π section

section and anti resonant circuit formed in the series arm of m-derived π section. We can select a frequency of resonance of these resonant circuits such that attenuation increases very rapidly at that frequency up to ∞. Consider m-derived low pass filter. Here if frequency of resonance is selected just above the cut-off frequency of the low pass filter, f_c, the shunt arm in T section acts as a short circuit at this frequency being series resonant circuit. It short circuits the transmission path. Hence output becomes zero and attenuation increases to ∞. In π section, at frequency which is just above f_c, series arm becomes open circuit being anti resonant circuit. Hence output becomes zero and attenuation increases to ∞.

Consider the m-derived T and π sections as shown in the Fig. 22 and Fig. 24. If we put m=1, both sections reduce to corresponding prototype sections. As m is a constant, for different values of m, we can design infinite m-derived sections for same f_c and z_0 specifications. The shunt arm of T section is a series combination of impedances in terms of Z_1 and Z_2 ; while series arm of π section is a parallel combination of impedances in terms of Z_1 and Z_2. But we know that Z_1 and Z_2 must be of different types. So to maintain this relationship in the shunt arm of T section and the series arm of π section, factor $1-m^2/4m$ must be positive which gives m always positive. As we have seen earlier, for m=1, we get original prototype filter sectio $0 < m<1$ of m must be always selected as

m Derived Low Pass Filter:

The m Derived Low Pass Filter T and π sections are as shown in the Fig. 23 (a) and (b) respectively.

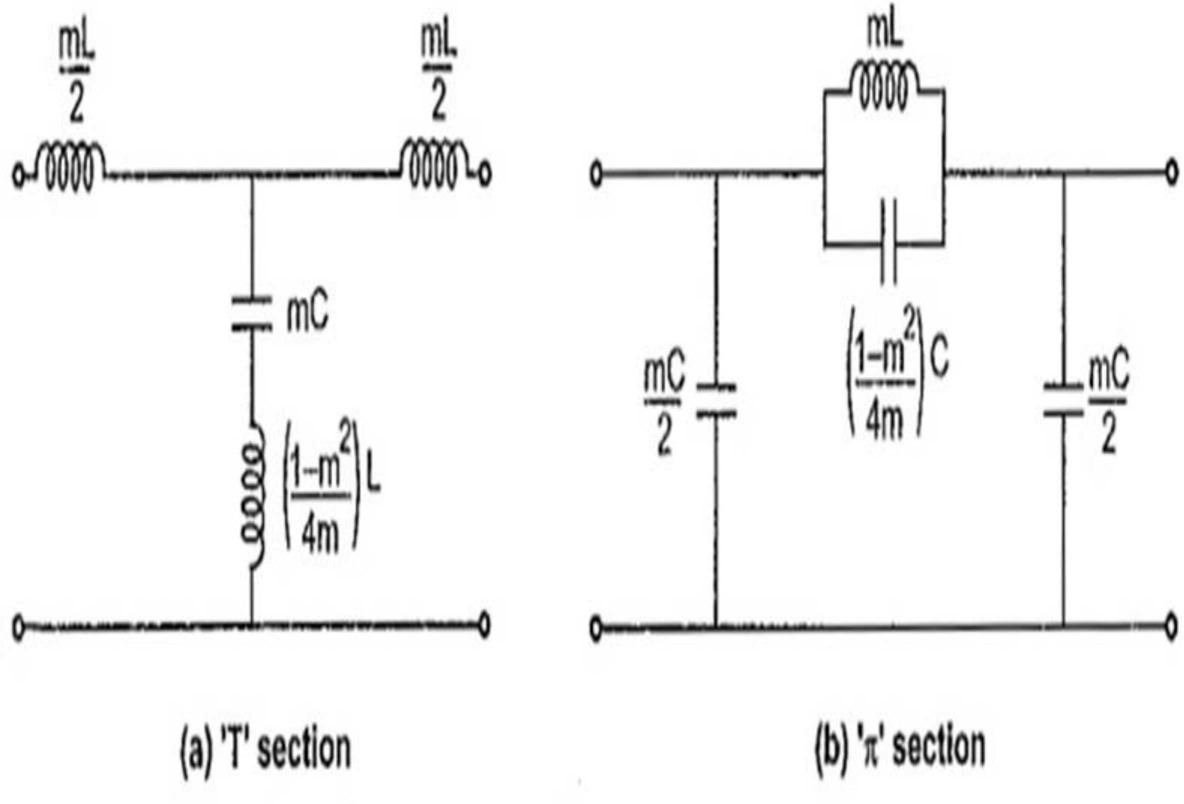


Fig 23 m-Derived Low Pass Filter

Consider that the shunt arm of T section resonates at the frequency of infinite attenuation i.e. f_∞, whic
cut-off frequency f_c. The frequency of resonance is given by,

$$f_\infty = \frac{1}{2\pi\sqrt{\left(\frac{1-m^2}{4m}\right)L(mC)}}$$

$$f_\infty = \frac{1}{\pi\sqrt{\left(1-m^2\right)LC}}$$

But for low pass filter, cut-off frequency is given by,

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

Substituting above value in equation for f_∞,

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}} \qquad \ldots(1)$$

Simplifying equation (1),

$$\sqrt{1-m^2} = \frac{f_c}{f_\infty}$$

$$\left(1-m^2\right) = \left(\frac{f_c}{f_\infty}\right)^2$$

$$m^2 = 1-\left(\frac{f_c}{f_\infty}\right)^2$$

$$m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2} \qquad \ldots (2)$$

Thus, equation (2) clearly indicates that if the cut-off frequency and frequency of infinite attenuation are specified the value of m can be easily evaluated.

Variations of Characteristic Impedance (Z_0) , Attenuation

Variations of Characteristic Impedance (Z_0) , Attenuation Constant (α) and Phase Constant (β) with Frequency

The variations of characteristic impedance (Z_0), attenuation constant (α) and phase constant (β) with frequency are as shown in the Fig. 2

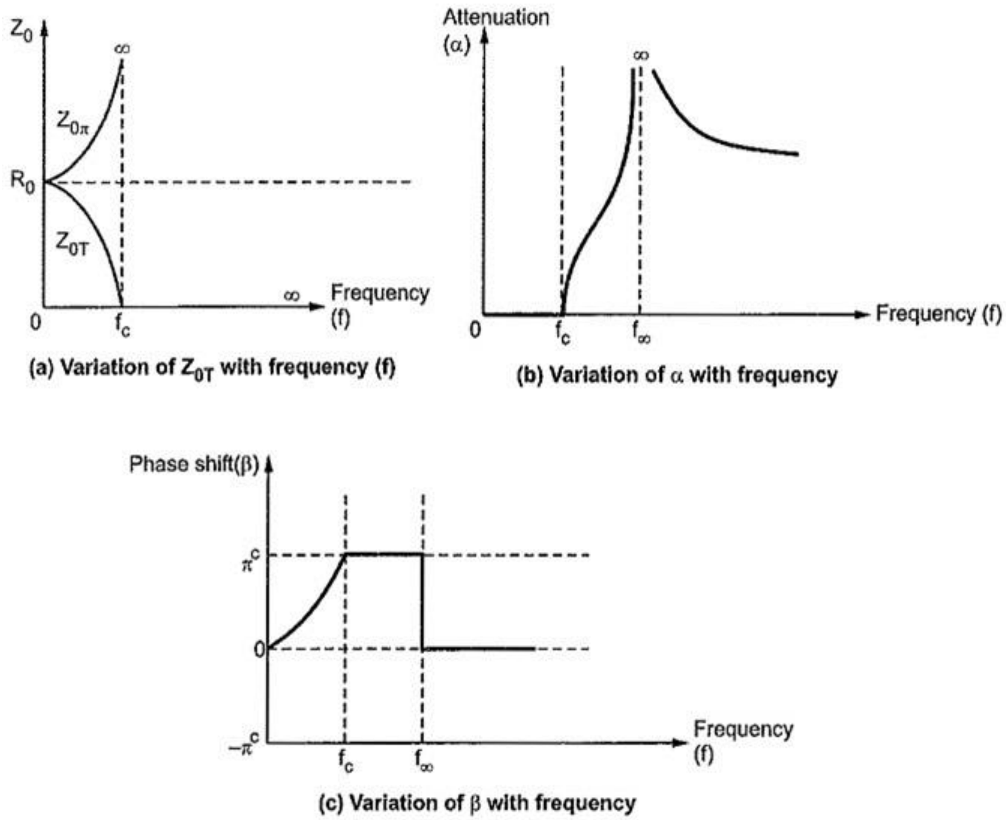


Fig 24

As the characteristic impedance Z_0 for prototype filter and m-derived section is same, the variation of Z_0 in m Derived Low Pass Filter section is similar to that in prototype filter section. The variation of attenuation constant over the attenuation band depends on types of the reactances. The attenuation constant a in attenuation band is given by,

$$\alpha = 2\cosh^{-1}\sqrt{\frac{Z_1}{4Z_2}} = 2\cosh^{-1}\left[\frac{m\left(\frac{f}{f_c}\right)}{\sqrt{1-\left(\frac{f}{f_\infty}\right)^2}}\right] \quad \text{for } f_c < f < f_\infty$$

$$\alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}} = 2\sinh^{-1}\left[\frac{m\left(\frac{f}{f_c}\right)}{\sqrt{\left(\frac{f}{f_\infty}\right)^2-1}}\right] \quad \text{for } f > f_\infty$$

The phase shift β in pass band is given by,

$$\beta = 2\sin^{-1}\sqrt{\frac{Z_1}{4Z_2}}$$

$$\beta = 2\sin^{-1}\left[\frac{m\left(\frac{f}{f_c}\right)}{\sqrt{1-\left(1-m^2\right)\left(\frac{f}{f_c}\right)^2}}\right]$$

As studied earlier, in stop band phase shift is always π^c. But in m Derived Low Pass Filter, above f_∞, the phase shift drops to 0 value as shunt arm becomes inductive above resonant frequency

m - Derived High Pass Filter:

The m Derived High Pass Filter T and π sections are as shown in the Fig. 25 (a) an

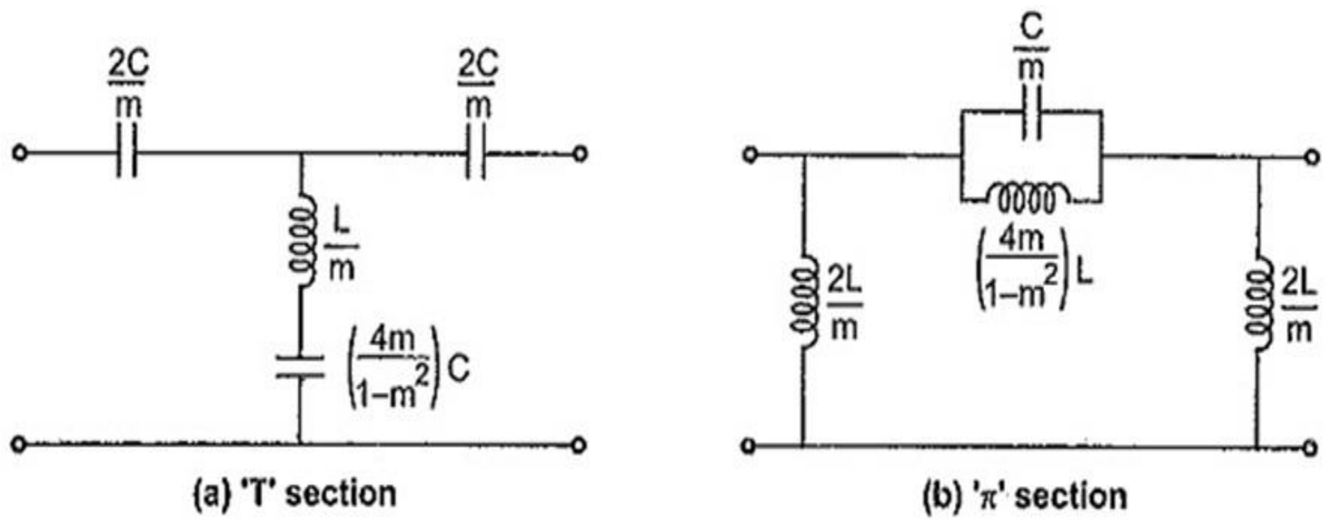


Fig 25 m-Derived high pass filter

Consider that the shunt arm of the T section resonates at a frequency of infinite attenuation i.e. f_∞ which is selected just below cut-off frequency f_c. The frequency of resonance is given by,

$$f_\infty = \frac{1}{2\pi\sqrt{\left(\frac{L}{m}\right)\left(\frac{4m}{1-m^2}\right)C}}$$

$$f_\infty = \frac{1}{2\pi\sqrt{\frac{4LC}{(1-m^2)}}}$$

$$f_\infty = \frac{\sqrt{(1-m^2)}}{4\pi\sqrt{LC}}$$

But for high pass filter, cut-off frequency is given by

$$4\pi\sqrt{LC}$$

Substituting value of f_c in the equation for f_∞,

$$\dots (1)$$

Simplifying equation (1) further,

$$\frac{}{f_c} = \sqrt{1-m}$$

$$\left(\frac{f_\infty}{f_c}\right)^2 = 1-m^2$$

$$m^2 = 1-\left(\frac{f_\infty}{f_c}\right)^2$$

$$m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2} \qquad \dots (2)$$

Variations of Characteristic Impedance (Z_0), Attenuation Constant (α) and Phase Shift (β) with Frequen
The variations of characteristic impedance (Z_0), attenuation constant (α) and phase shift (β) with frequ
Respectively

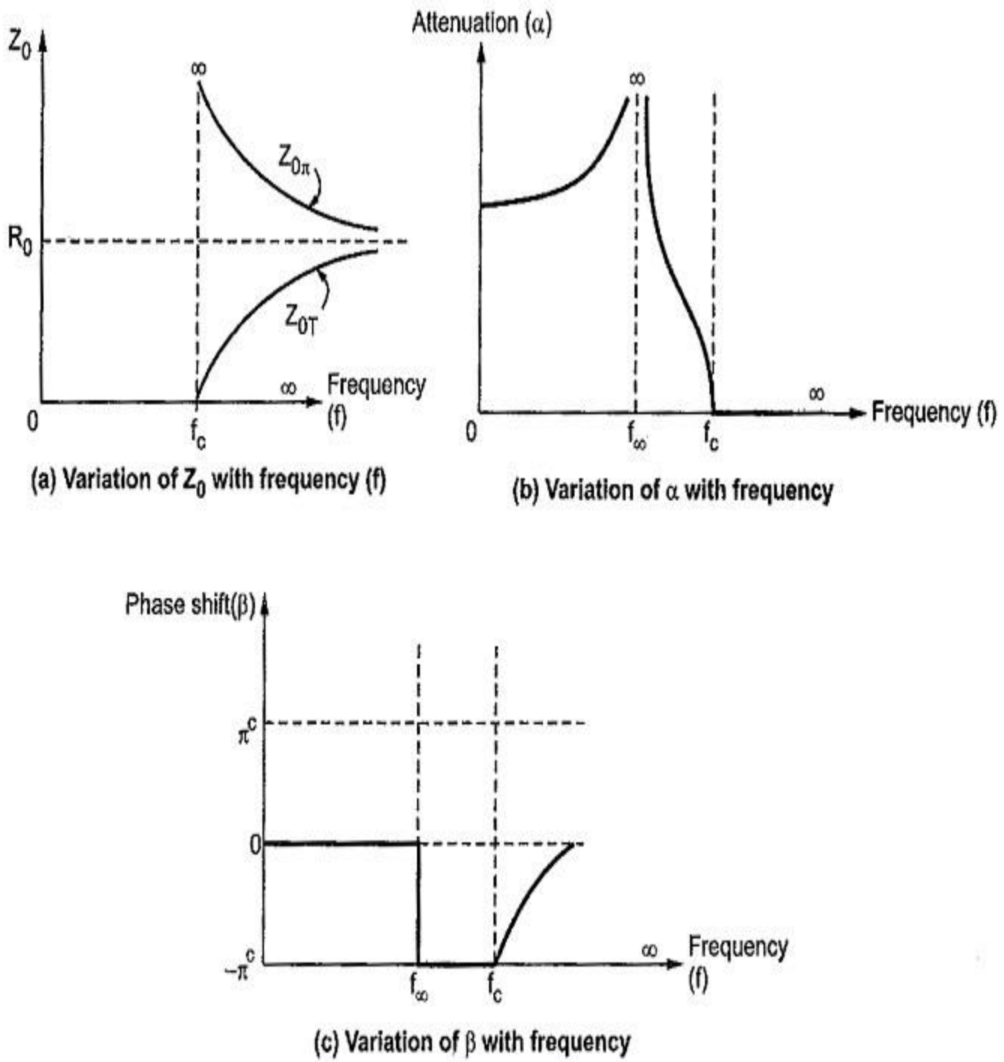


Fig 26

As the characteristic impedance for both the sections i.e. prototype section and m Derived High Pass Filter section, the variation of Z_0 in m-derived section is sim the prototype filter section. The attenuation constant α in the attenuation band is

$$\alpha = 2\cosh^{-1}\left[\frac{m\left(\frac{f_c}{f}\right)}{\sqrt{1-\left(\frac{f_\infty}{f}\right)^2}}\right] \text{ for } f_\infty < f < f_c$$

$$= 2\sinh^{-1}\left[\frac{m\left(\frac{f_c}{f}\right)}{\sqrt{\left(\frac{f_\infty}{f}\right)^2-1}}\right] \text{ for } f < f_\infty$$

The phase shift β in the pass band is given by

$$\beta = 2\sin^{-1}\left[\frac{m\left(\frac{f_c}{f}\right)}{\sqrt{1-\left(1-m^2\right)\left(\frac{f_c}{f}\right)^2}}\right]$$

Composite Filters in Network Analysis – In prototype filter sections, the attenuation characteristic is not very sharp in the attenuation band as it is expected. This drawback can be overcome by using m-derived filter sections which are derived from respective prototype filter sections. But it is observed that in the stop band attenuation drastically reduces after f_∞ in low pass section and before f_∞ in high pass section. This drawback of m-derived filter can be overcome by connecting number of sections including prototype sections and m-derived sections with terminating half sections. Such a combination of different sections is called **composite filter**.

The comparison of attenuation constant variations in low pass prototype section, m-derived section and com

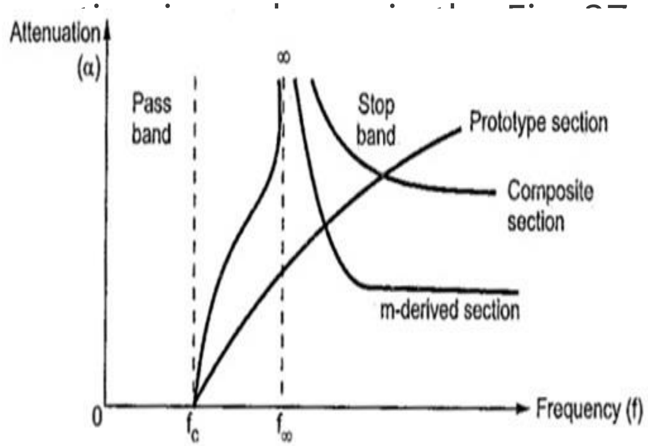


Fig. 27 Variation of attenuation constant in prototype, m-derived, and composite section

impedance are the two important design specifications. The number of various sections in the composite filter totally depends on the attenuation characteristics required. If it is required that the attenuation should rise sharply in the attenuation band, we must select at least one m-derived section with low value of m.

In general, for lower values of m, attenuation at cut off rises rapidly. The typical value of m for such attenuation at cut off is m= 0.3 to 0.35. If it is required to maintain this attenuation at a high value in attenuation band, we must connect either a prototype section or another m-derived section with comparatively larger value of m.

If required both the sections can be used in the composite filter. To have proper impedance matching and constant characteristic impedance throughout pass band, we must connect).6.

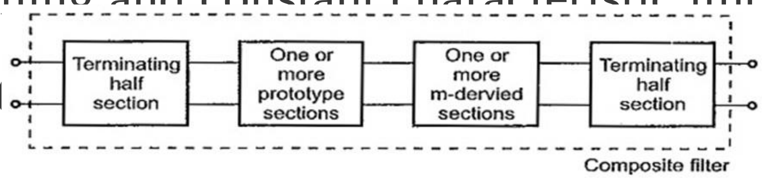


Fig. 28 Composite filter

eral block schematic of the composite filter is as shown in the Fig. 9.45.

Active Filters :

Active Filters – An electric filter is often a frequency selective circuit that passes a specified band of frequencies and blocks or attenuates signals of frequencies outside this band. Filters may be classified in a number of ways as follows.

1. **Analog or digital filters**
2. **Passive or active filters**
3. **Audio (AF) or radio (RF) filters**

Analog filters are designed to process analog signals using analog techniques, while digital filters process analog signals using digital techniques. Depending on the type of elements used in their construction, filters may be classified as active or passive.
The elements used in passive filters are R, C and L. Active filters, on the other hand, employ transistors or op-amps in addition to resistors and capacitors. The type of element used dictates the operating frequency range of the filter.

For example, RC filters are commonly used for audio or low frequency operation, whereas LC filters or crystal filters are employed at RF or high frequency. Because of their high Q value (figure of merit), the crystals provide stable operation at high frequency. Inductors are not used in the audio range

1.Gain and frequency adjustment flexibility
Since the op-amp is capable of providing a gain, the input signal is not attenuated, as in a passive filter. Also, an active filter is easier to tune.

2.No loading problem
Because of the high input resistance and low output resistance of the op-amp, the active filter does not cause loading of the source or load.

3.Cost
Active filters are typically more economical than passive filters. This is because of the variety of cheaper op-amps available, and the absence of inductors.

Although active filters are most extensively used in the field of communications and signal processing, they are employed in one form or another in almost all sophisticated electronic systems, radio, TV, telephones, radar, space satellites and biomedical equipment.

The most commonly used Active Filters Classification are as follows.

1. Low pass
2. High pass
3. Band pass
4. Band stop
5. All pass

Each of these filters uses an opamp as the active element and R, C as the passive element.

resonators they use makes them ideal for use as the primary band pass RF filter in high performance radio communications receivers. As a result of this there are a number of circuits that have been used to provide the required level of selectivity and performance over the years. These include the single crystal filter, the half lattice crystal filter and the ladder filter.

Single crystal filter

The simplest crystal filter employs a single crystal. This type of RF filter was developed in the 1930s and was used in early receivers dating from before the 1960s but is rarely used today. Although it employs the very high Q of the crystal, its response is asymmetric and it is too narrow for most applications, having a bandwidth of a hundred Hz or less.

In the circuit there is a variable capacitor that is used to compensate for the parasitic capacitance in the crystal. This capacitor was normally included as a front panel control.

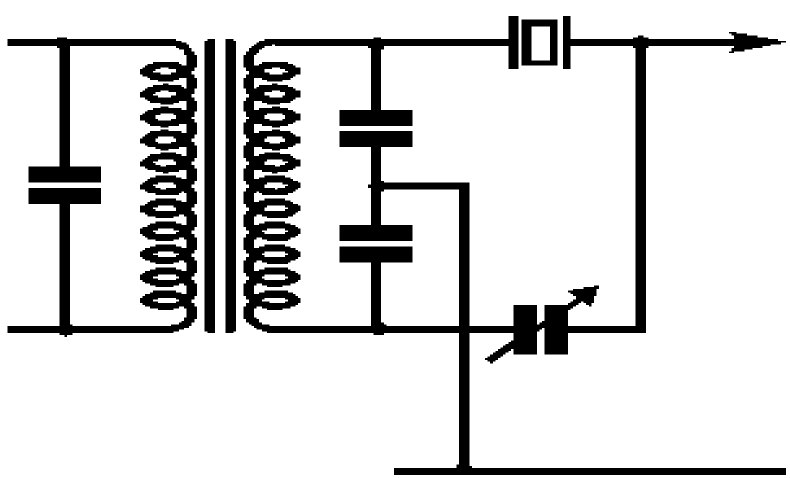


Diagram of filter using a single quartz crystal

Half Lattice crystal filter

This form of band pass RF filter provided a significant improvement in performance over the single. In this configuration the parasitic capacitances of each of the crystals cancel each other out and enable the circuit to operate satisfactorily. By adopting a slightly different frequency for the crystals, a wider bandwidth is obtained. However the slope response outside the required pass band falls away quickly, enabling high levels of out of band rejection to be obtained. Typically the parallel resonant frequency of one crystal is designed to be equal to the series resonant frequency of the other.

Despite the fact that the half lattice crystal filter can offer a much flatter in-band response there is still some ripple. This results from the fact that the two crystals have different resonant frequencies. The response has a small peak at either side of the centre frequency and a small dip in the middle. As a rough rule of thumb it is found that the 3 dB bandwidth of the RF filter is about 1.5 times the frequency difference between the two resonant frequencies. It is also found that for optimum performance the matching of the filter is very important. To achieve this, matching resistors are often placed on the input and output. If the filter is not properly matched then it is found that there will be more in-band ripple and the ultimate rejection may not be as

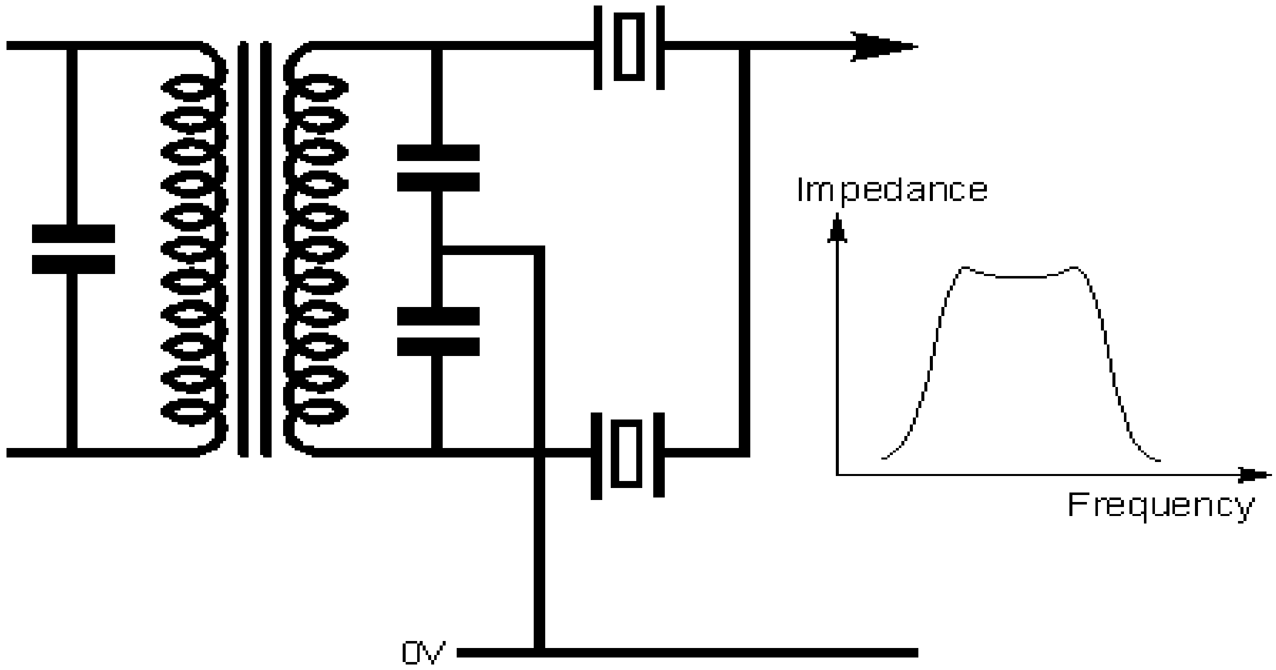
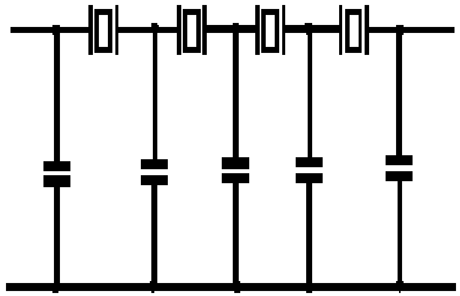


Diagram of half lattice crystal filter

A two pole filter (i.e. one with two crystals) is not normally adequate to meet many requirements. The shape factor can be greatly improved by adding further sections. Typically ultimate rejections of 70 dB and more are required in a receiver. As a rough guide a two pole filter will generally give a rejection of around 20 dB; a four pole filter, 50 dB; a six pole filter, 70 dB; and an eight pole one 90 dB.

Crystal ladder filter

For many years the half lattice filter was possibly the most popular format used for crystal filters. More recently the ladder topology has gained considerable acceptance. In this form of crystal pass band filter all the resonators have the same frequency, and the inter-resonator coupling is provided by the capacitors placed between the resonators with the other termination connected to earth.



Four pole ladder crystal filter

Quartz crystal filter design parameters

When a quartz crystal filter is designed factors such as the input and output impedance as well as bandwidth, crystal Q and many other factors need to be taken into account.

Some of the chief factors are obviously the bandwidth, shape fact, and ultimate cutoff. Although it is very much a simplification, these factors are dependent upon the number of poles (equivalent to the number of crystals), their Q value, and their individual frequencies.

Crystal filters are widely used in many radio communications receiver applications. Here these filters are able to provide very high levels of performance and at a cost which is very reasonable for the performance that is given. These RF filters may be made in a variety of formats according to the applications and the performance needed. While these RF filters can be made from discrete components, ready manufactured crystal filters are normally bought, either off the shelf, of made to a given specification.

Thank You